CHAPTER 1
Equations, Inequalities, and Mathematical Modeling

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Section 1.1 Graphs of Equations

1. solution or solution point

2. graph

3. intercepts

4. y-axis

5. origin

6. numerical

7. Two other approaches to solve problems mathematically are algebraic and graphical.

8. Let (x, y) be any point on the circle. The distance between the center (h, k) and (x, y) is the radius r. So, $\sqrt{(x-h)^2 + (y-k)^2} = r$.

9. (a) $(2, 0): (2)^2 - 3(2) + 2 \stackrel{?}{=} 0$

$$4 - 6 + 2 \stackrel{?}{=} 0$$
$$0 = 0$$

Yes, the point *is* on the graph.

(b) $(-2, 8): (-2)^2 - 3(-2) + 2 \stackrel{?}{=} 8$

$$4 + 6 + 2 \stackrel{?}{=} 8$$
$$12 \neq 8$$

No, the point *is not* on the graph.

10. (a) $(0, 2): 2 \stackrel{?}{=} \sqrt{0 + 4}$

$$2 = 2$$

Yes, the point *is* on the graph.

(b) $(5, 3): 3 \stackrel{?}{=} \sqrt{5 + 4}$

$$3 \stackrel{?}{=} \sqrt{9}$$
$$3 = 3$$

Yes, the point *is* on the graph.

11. (a) $(1, 5): 5 \stackrel{?}{=} 4 - |1 - 2|$

$$5 \stackrel{?}{=} 4 - 1$$
$$5 \neq 3$$

No, the point *is not* on the graph.

(b) $(6, 0): 0 \stackrel{?}{=} 4 - |6 - 2|$

$$0 \stackrel{?}{=} 4 - 4$$
$$0 = 0$$

Yes, the point *is* on the graph.

12. (a) $(6, 0): 2(6)^2 + 5(0)^2 \stackrel{?}{=} 8$

$$2(36) + 5(0) \stackrel{?}{=} 8$$
$$72 + 0 \stackrel{?}{=} 8$$
$$72 \neq 8$$

No, the point *is not* on the graph.

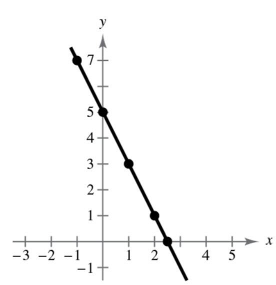
(b) $(0, 4): 2(0)^2 + 5(4)^2 \stackrel{?}{=} 8$

$$2(0) + 5(16) \stackrel{?}{=} 8$$
$$0 + 80 \stackrel{?}{=} 8$$
$$80 \neq 8$$

No, the point *is not* on the graph.

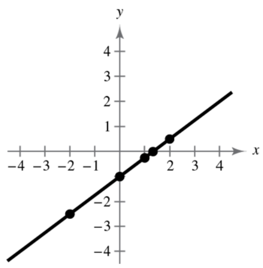
13. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y	7	5	3	1	0
(x, y)	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$\left(\frac{5}{2}, 0\right)$



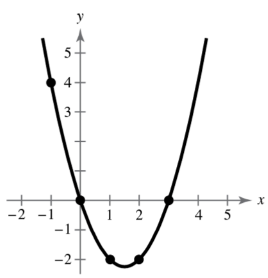
14. $y = \frac{3}{4}x - 1$

x	-2	0	1	$\frac{4}{3}$	2
y	$-\frac{5}{2}$	-1	$-\frac{1}{4}$	0	$\frac{1}{2}$
(x, y)	$\left(-2, -\frac{5}{2}\right)$	$(0, -1)$	$\left(1, -\frac{1}{4}\right)$	$\left(\frac{4}{3}, 0\right)$	$\left(2, \frac{1}{2}\right)$



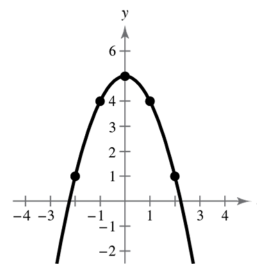
15. $y = x^2 - 3x$

x	-1	0	1	2	3
y	4	0	-2	-2	0
(x, y)	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$



16. $y = 5 - x^2$

x	-2	-1	0	1	2
y	1	4	5	4	1
x, y	$(-2, 1)$	$(-1, 4)$	$(0, 5)$	$(1, 4)$	$(2, 1)$



17. x-intercept: $(-2, 0)$

y-intercept: $(0, 2)$

18. x-intercept: $(4, 0)$

y-intercepts: $(0, \pm 2)$

19. x-intercept: $(3, 0)$

y-intercept: $(0, 9)$

20. x-intercepts: $(\pm 2, 0)$

y-intercept: $(0, 16)$

21. x-intercept: $(1, 0)$

y-intercept: $(0, 2)$

22. x-intercepts: $(0, 0), (\pm 2, 0)$

y-intercept: $(0, 0)$

23. $x^2 - y = 0$

$(-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow$ y-axis symmetry

$x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No x-axis symmetry

$(-x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No origin symmetry

24. $x - y^2 = 0$

$(-x) - y^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No y-axis symmetry

$x - (-y)^2 = 0 \Rightarrow x - y^2 = 0 \Rightarrow$ x-axis symmetry

$(-x) - (-y)^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No origin symmetry

25. $y = x^3$

$y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow$ No y-axis symmetry

$-y = x^3 \Rightarrow y = -x^3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow$ Origin symmetry

26. $y = x^4 - x^2 + 3$

$y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = x^4 - x^2 + 3 \Rightarrow$ y-axis symmetry

$-y = x^4 - x^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No origin symmetry

27. $y = \dfrac{x}{x^2 + 1}$

$y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No y-axis symmetry

$-y = \dfrac{x}{x^2 + 1} \Rightarrow y = \dfrac{-x}{x^2 + 1} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{-x}{(-x)^2 + 1} \Rightarrow -y = \dfrac{-x}{x^2 + 1} \Rightarrow y = \dfrac{x}{x^2 + 1} \Rightarrow$ Origin symmetry

28. $y = \dfrac{1}{1 + x^2}$

$y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{1}{1 + x^2} \Rightarrow$ y-axis symmetry

$-y = \dfrac{1}{1 + x^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{1}{1 + (-x)^2} \Rightarrow y = \dfrac{-1}{1 + x^2} \Rightarrow$ No origin symmetry

29. $xy^2 + 10 = 0$

$(-x)y^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No y-axis symmetry

$x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow$ x-axis symmetry

$(-x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow$ No origin symmetry

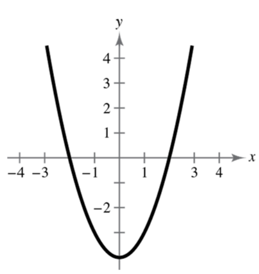
30. $xy = 4$

$(-x)y = 4 \Rightarrow xy = -4 \Rightarrow$ No y-axis symmetry

$x(-y) = 4 \Rightarrow xy = -4 \Rightarrow$ No x-axis symmetry

$(-x)(-y) = 4 \Rightarrow xy = 4 \Rightarrow$ Origin symmetry

31.



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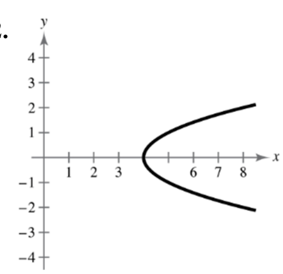

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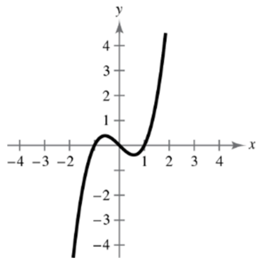

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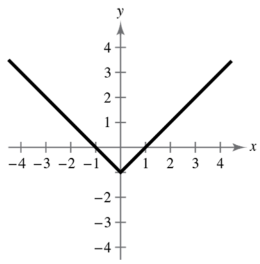
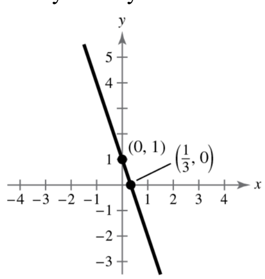


35. $y = -3x + 1$

x-intercept: $\left(\frac{1}{3}, 0\right)$

y-intercept: $(0, 1)$

No symmetry

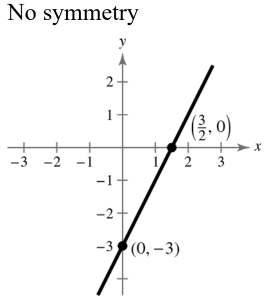


36. $y = 2x - 3$

x-intercept: $\left(\frac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

No symmetry



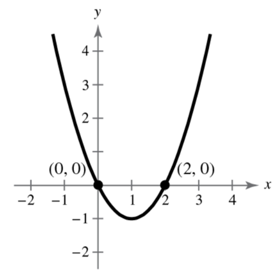
37. $y = x^2 - 2x$

x-intercepts: $(0, 0), (2, 0)$

y-intercept: $(0, 0)$

No symmetry

x	−1	0	1	2	3
y	3	0	−1	0	3

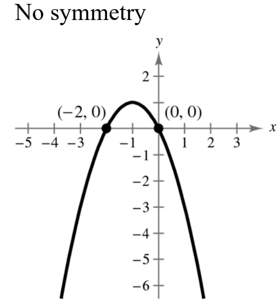


38. $y = -x^2 - 2x$

x-intercepts: $(-2, 0), (0, 0)$

y-intercept: $(0, 0)$

No symmetry



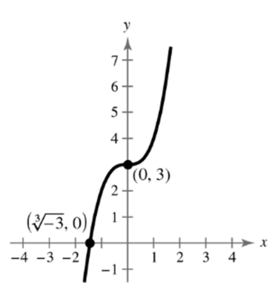
39. $y = x^3 + 3$

x-intercept: $\left(\sqrt[3]{-3}, 0\right)$

y-intercept: $(0, 3)$

No symmetry

x	−2	−1	0	1	2
y	−5	2	3	4	11

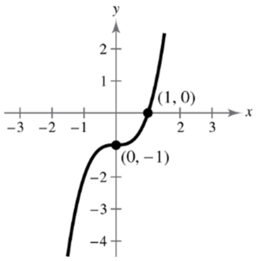


40. $y = x^3 - 1$

x-intercept: $(1, 0)$

y-intercept: $(0, -1)$

No symmetry



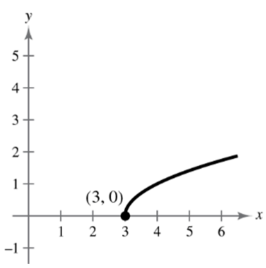
41. $y = \sqrt{x - 3}$

x-intercept: $(3, 0)$

y-intercept: none

No symmetry

x	3	4	7	12
y	0	1	2	3

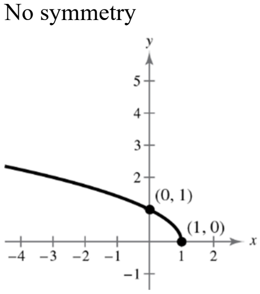


42. $y = \sqrt{1 - x}$

x-intercept: $(1, 0)$

y-intercept: $(0, 1)$

No symmetry



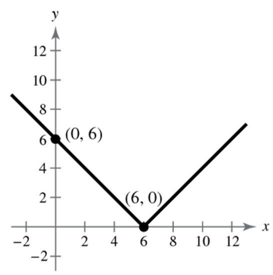
43. $y = |x - 6|$

x-intercept: $(6, 0)$

y-intercept: $(0, 6)$

No symmetry

x	−2	0	2	4	6	8	10
y	8	6	4	2	0	2	4

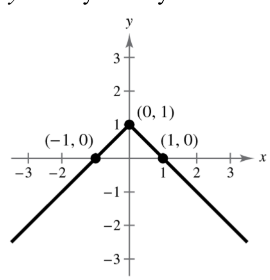


44. $y = 1 - |x|$

x-intercepts: $(1, 0), (-1, 0)$

y-intercept: $(0, 1)$

y-axis symmetry



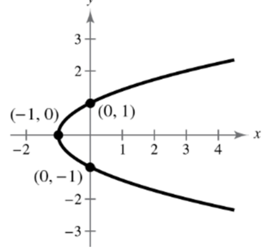
45. $x = y^2 - 1$

x-intercept: $(-1, 0)$

y-intercepts: $(0, -1), (0, 1)$

x-axis symmetry

x	−1	0	3
y	0	±1	±2

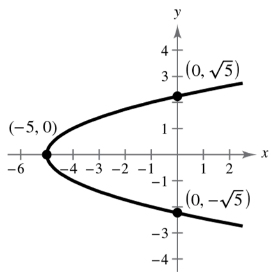


46. $x = y^2 - 5$

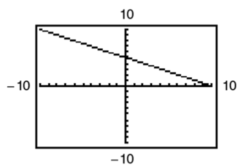
x-intercept: $(-5, 0)$

y-intercepts: $(0, \sqrt{5}), (0, -\sqrt{5})$

x-axis symmetry

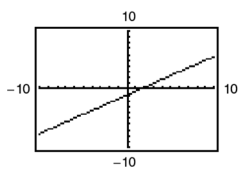


47. $y = 5 - \frac{1}{2}x$



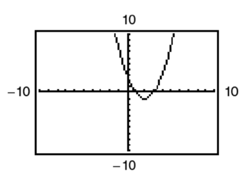
Intercepts: $(10, 0), (0, 5)$

48. $y = \frac{2}{3}x - 1$



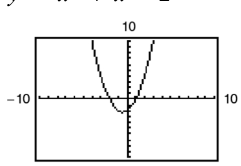
Intercepts: $(0, -1), \left(\frac{3}{2}, 0\right)$

49. $y = x^2 - 4x + 3$



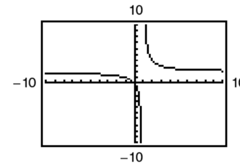
Intercepts: $(3, 0), (1, 0), (0, 3)$

50. $y = x^2 + x - 2$



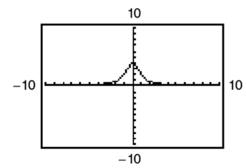
Intercepts: $(-2, 0), (1, 0), (0, -2)$

51. $y = \dfrac{2x}{x - 1}$



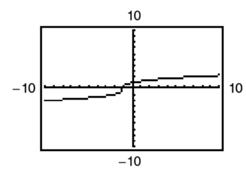
Intercept: $(0, 0)$

52. $y = \dfrac{4}{x^2 + 1}$



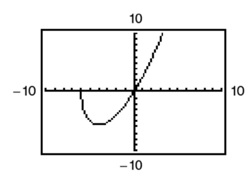
Intercept: $(0, 4)$

53. $y = \sqrt[3]{x + 1}$



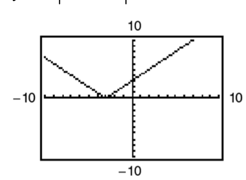
Intercepts: $(-1, 0), (0, 1)$

54. $y = x\sqrt{x + 6}$



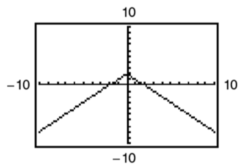
Intercepts: $(0, 0), (-6, 0)$

55. $y = |x + 3|$



Intercepts: $(-3, 0), (0, 3)$

56. $y = 2 - |x|$



Intercepts: $(\pm 2, 0), (0, 2)$

57. Center: $(0, 0)$; Radius: 3

$$(x - 0)^2 + (y - 0)^2 = 3^2$$
$$x^2 + y^2 = 9$$

58. Center: $(0, 0)$; Radius: 7

$$(x - 0)^2 + (y - 0)^2 = 7^2$$
$$x^2 + y^2 = 49$$

59. Center: $(-4, 5)$; Radius: 2

$$(x - h)^2 + (y - k)^2 = r^2$$
$$[x - (-4)]^2 + [y - 5]^2 = 2^2$$
$$(x + 4)^2 + (y - 5)^2 = 4$$

60. Center: $(1, -3)$; Radius: $\sqrt{11}$

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 1)^2 + [y - (-3)]^2 = \sqrt{11}^2$$
$$(x - 1)^2 + (y + 3)^2 = 11$$

61. Center: $(3, 8)$; Solution point: $(-9, 13)$

$$\begin{aligned} r &= \sqrt{(x - h)^2 + (y - k)^2} \\ &= \sqrt{(-9 - 3)^2 + (13 - 8)^2} \\ &= \sqrt{(-12)^2 + (5)^2} \\ &= \sqrt{144 + 25} \\ &= \sqrt{169} \\ &= 13 \end{aligned}$$

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 3)^2 + (y - 8)^2 = 13^2$$
$$(x - 3)^2 + (y - 8)^2 = 169$$

62. Center: $(-2, -6)$; Solution point: $(1, -10)$

$$\begin{aligned} r &= \sqrt{(x - h)^2 + (y - k)^2} \\ &= \sqrt{[1 - (-2)]^2 + [-10 - (-6)]^2} \\ &= \sqrt{(3)^2 + (-4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$(x - h)^2 + (y - k)^2 = r^2$$
$$[x - (-2)]^2 + [y - (-6)]^2 = 5^2$$
$$(x + 2)^2 + (y + 6)^2 = 25$$

63. Endpoints of a diameter: $(3, 2), (-9, -8)$

$$\begin{aligned} r &= \frac{1}{2}\sqrt{(-9 - 3)^2 + (-8 - 2)^2} \\ &= \frac{1}{2}\sqrt{(-12)^2 + (-10)^2} \\ &= \frac{1}{2}\sqrt{144 + 100} \\ &= \frac{1}{2}\sqrt{244} = \frac{1}{2}(2)\sqrt{61} = \sqrt{61} \end{aligned}$$

$$(h, k): \left(\frac{3 + (-9)}{2} \cdot \frac{2 + (-8)}{2}\right) = \left(\frac{-6}{2} \cdot \frac{-6}{2}\right) = (-3, -3)$$

$$(x - h)^2 + (y - k)^2 = r^2$$
$$[x - (-3)]^2 + [y - (-3)]^2 = \left(\sqrt{61}\right)^2$$
$$(x + 3)^2 + (y + 3)^2 = 61$$

64. Endpoints of a diameter: $(11, -5), (3, 15)$

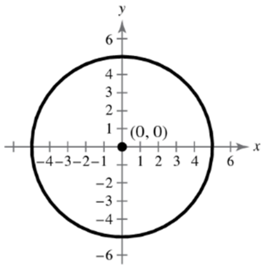
$$\begin{aligned} r &= \frac{1}{2}\sqrt{(3 - 11)^2 + [15 - (-5)]^2} \\ &= \frac{1}{2}\sqrt{(-8)^2 + (20)^2} \\ &= \frac{1}{2}\sqrt{64 + 400} \\ &= \frac{1}{2}\sqrt{464} = \frac{1}{2}(4)\sqrt{29} = 2\sqrt{29} \end{aligned}$$

$$(h, k): \left(\frac{11 + 3}{2} \cdot \frac{-5 + 15}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$$

$$(x - h)^2 + (y - k)^2 = r^2$$
$$(x - 7)^2 + (y - 5)^2 = \left(2\sqrt{29}\right)^2$$
$$(x - 7)^2 + (y - 5)^2 = 116$$

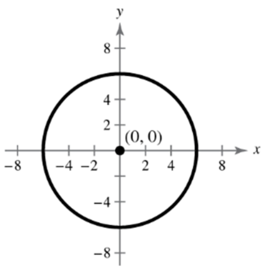
65. $x^2 + y^2 = 25$

Center: $(0, 0)$, Radius: 5



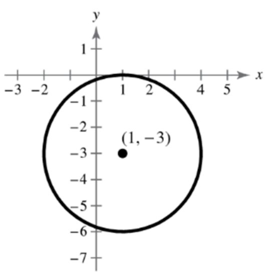
66. $x^2 + y^2 = 36$

Center: $(0, 0)$, Radius: 6



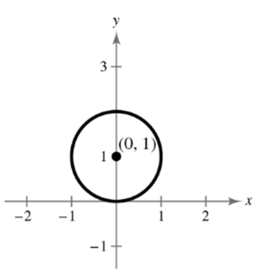
67. $(x - 1)^2 + (y + 3)^2 = 9$

Center: $(1, -3)$, Radius: 3



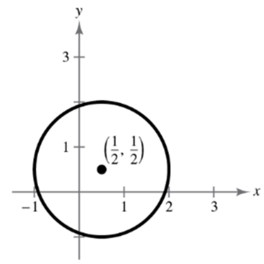
68. $x^2 + (y - 1)^2 = 1$

Center: $(0, 1)$, Radius: 1



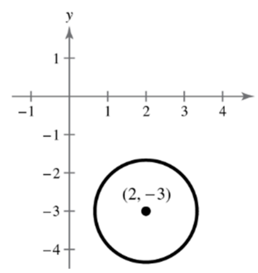
69. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

Center: $\left(\frac{1}{2}, \frac{1}{2}\right)$, Radius: $\frac{3}{2}$

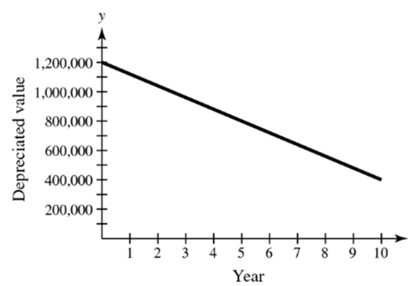


70. $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

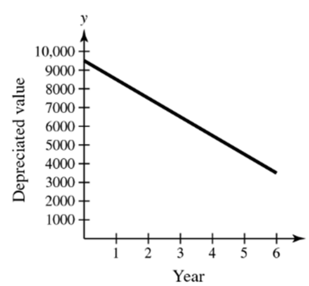
Center: $(2, -3)$, Radius: $\frac{4}{3}$



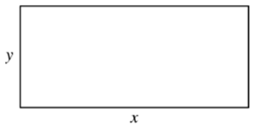
71. $y = 1{,}200{,}000 - 80{,}000t,\ 0 \le t \le 10$



72. $y = 9500 - 1000t,\ 0 \le t \le 6$



73. (a)



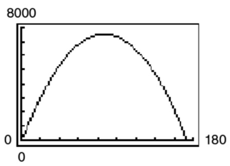
(b) $2x + 2y = \frac{1040}{3}$

$2y = \frac{1040}{3} - 2x$

$y = \frac{520}{3} - x$

$A = xy = x\left(\frac{520}{3} - x\right)$

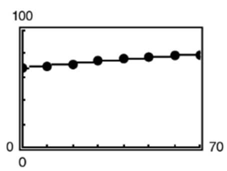
(c)



(d) When $x = y = 86\frac{2}{3}$ yards, the area is a maximum of $7511\frac{1}{9}$ square yards.

(e) A regulation NFL playing field is 120 yards long and $53\frac{1}{3}$ yards wide. The actual area is 6400 square yards.

74. (a)



The model fits the data well.

Each data value is close to the graph of the model.

(b) Using the graph or the model (with $t = 40$),

The life expectancy is approximately 75.2 years.

(c) The model and the line $y = 70.1$ intersect at $t = 11.1$, corresponding to the year 1961.

Algebraically, $\dfrac{68.0 + 0.33(11.1)}{1 + 0.002(11.1)} \approx 70.1.$

(d) The y-intercept is $(0, 68.0)$. In 1950 $(t = 0)$, the life expectancy of a child (at birth) was 68.0 years.

(e) Answers will vary.

75. False. The line $y = x$ is symmetric with respect to the origin.

76. False. The line $y = 0$ has infinitely many x-intercepts.

77. The test is for symmetry with respect to the x-axis. The statement should read: The graph of $x = 3y^2$ is symmetric with respect to the x-axis because

78. x-axis symmetry:

$$x^2 + y^2 = 1$$
$$x^2 + (-y)^2 = 1$$
$$x^2 + y^2 = 1$$

y-axis symmetry:

$$x^2 + y^2 = 1$$
$$(-x)^2 + y^2 = 1$$
$$x^2 + y^2 = 1$$

Origin symmetry:

$$x^2 + y^2 = 1$$
$$(-x)^2 + (-y)^2 = 1$$
$$x^2 + y^2 = 1$$

So, the graph of the equation is symmetric with respect to x-axis, y-axis, and origin.

79. $y = ax^2 + bx^3$

(a) $y = a(-x)^2 + b(-x)^3$

$= ax^2 - bx^3$

To be symmetric with respect to the y-axis; a can be any non-zero real number, b must be zero.

Sample answer: $a = 1, b = 0$

(b) $-y = a(-x)^2 + b(-x)^3$

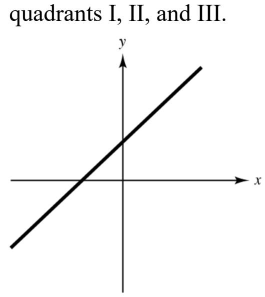
$-y = ax^2 - bx^3$

$y = -ax^2 + bx^3$

To be symmetric with respect to the origin; a must be zero, b can be any non-zero real number.

Sample answer: $a = 0, b = 1$

80. The line would rise from left to right, passing through quadrants I, II, and III.



81. $3(7x + 1) = 3(7x) + 3(1) = 21x + 3$

82. $5(x - 6) = 5(x) - 5(6) = 5x - 30$

83. $6(x - 1) + 4 = 6(x) - 6(1) = 6x - 6 + 4 = 6x - 2$

84. $4(x + 2) - 12 = 4(x) + 4(2) = 4x + 8 - 12$

$= 4x - 4$

85. The least common denominator is $3(4) = 12$.

86. The least common denominator is 9.

87. The least common denominator is $x - 4$.

88. The least common denominator is $x^2 - 4 = (x + 2)(x - 2)$.

89. $$\begin{aligned} 7\sqrt{72} - 5\sqrt{18} &= 7\sqrt{2 \cdot 36} - 5\sqrt{2 \cdot 9} \\ &= 7(6)\sqrt{2} - 5(3)\sqrt{2} \\ &= (42 - 15)\sqrt{2} \\ &= 27\sqrt{2} \end{aligned}$$

90. $$\begin{aligned} -10\sqrt{25y} - \sqrt{y} &= (-10)(5)\left(\sqrt{y} - \sqrt{y}\right) \\ &= -50\left(\sqrt{y} - \sqrt{y}\right) \\ &= -51\sqrt{y} \end{aligned}$$

91. $7^{3/2} \cdot 7^{11/2} = 7^{3/2+11/2} = 7^7 = 823{,}543$

92. $\dfrac{10^{17/4}}{10^{5/4}} = 10^{17/4-5/4} = 10^{12/4} = 10^3 = 1000$

93. $(9x - 4) + \left(2x^2 - x + 15\right) = 2x^2 + 8x + 11$

94. $$\begin{aligned} 4x\left(11 - x + 3x^2\right) &= 44x - 4x^2 + 12x^3 \\ &= 12x^3 - 4x^2 + 44x \end{aligned}$$

95. $$\begin{aligned} (2x + 9)(x - 7) &= 2x^2 + 9x - 14x - 63 \\ &= 2x^2 - 5x - 63 \end{aligned}$$

96. $$\begin{aligned} \left(3x^2 - 5\right)\left(-x^2 + 1\right) &= -3x^4 + 5x^2 + 3x^2 - 5 \\ &= -3x^4 + 8x^2 - 5 \end{aligned}$$

Section 1.2 Linear Equations in One Variable

1. equation

2. identities; conditional; contradictions

3. $ax + b = 0$

4. equivalent

5. rational

6. extraneous

7. Yes. $8 = x - 3$ and $x = 11$ are equivalent equations.

8. $4\left(\dfrac{x}{2} + 1\right) = 4\left(\dfrac{1}{4}\right) \rightarrow 2x + 4 = 1$

9. The equation $3(x - 1) = 3x - 3$ is an *identity* by the Distributive Property. The equation is true for all real values of x.

10. The equation $2(x + 1) = 2x - 1$ is a *contradiction*. There are no real values of x for which the equation is true.

11. The equation $2(x - 1) = 3x + 1$ is a *conditional equation*. The only value in the domain that satisfies the equation is $x = -3$.

12. The equation $4(x + 2) = 2x + 2$ is a *conditional equation*. The only value in the domain that satisfies the equation is $x = -3$.

13. The equation $3(x + 2) = 3x + 2$ is a *contradiction*. There are no real values of x for which the equation is true.

14. The equation $5(x + 2) = 5x + 10$ is an *identity* by the Distributive Property. The equation is true for all real values of x.

15. The equation $2(x + 3) - 5 = 2x + 1$ is an *identity* by simplification. The equation is true for all real values of x.

16. The equation $3(x - 1) + 2 = 4x - 2$ is a *conditional equation*. The only value in the domain that satisfies the equation is $x = 1$.

17. $$\begin{aligned} 2x + 11 &= 15 \\ 2x + 11 - 11 &= 15 - 11 \\ 2x &= 4 \\ \frac{2x}{2} &= \frac{4}{2} \\ x &= 2 \end{aligned}$$

18. $$\begin{aligned} 7x + 2 &= 23 \\ 7x + 2 - 2 &= 23 - 2 \\ 7x &= 21 \\ \frac{7x}{7} &= \frac{21}{7} \\ x &= 3 \end{aligned}$$

19. $$\begin{aligned} 7 - 2x &= 25 \\ 7 - 7 - 2x &= 25 - 7 \\ -2x &= 18 \\ \frac{-2x}{-2} &= \frac{18}{-2} \\ x &= -9 \end{aligned}$$

20.
$$\begin{aligned} 7 - x &= 19 \\ 7 - x + x &= 19 + x \\ 7 &= 19 + x \\ 7 - 19 &= 19 + x - 19 \\ -12 &= x \end{aligned}$$

21.
$$\begin{aligned} 3x - 5 &= 2x + 7 \\ 3x - 2x - 5 &= 2x - 2x + 7 \\ x - 5 &= 7 \\ x - 5 + 5 &= 7 + 5 \\ x &= 12 \end{aligned}$$

22.
$$\begin{aligned} 5x + 3 &= 6 - 2x \\ 5x + 2x + 3 &= 6 - 2x + 2x \\ 7x + 3 &= 6 \\ 7x + 3 - 3 &= 6 - 3 \\ 7x &= 3 \\ \frac{7x}{7} &= \frac{3}{7} \\ x &= \frac{3}{7} \end{aligned}$$

23.
$$\begin{aligned} 4y + 2 - 5y &= 7 - 6y \\ 4y - 5y + 2 &= 7 - 6y \\ -y + 2 &= 7 - 6y \\ -y + 6y + 2 &= 7 - 6y + 6y \\ 5y + 2 &= 7 \\ 5y + 2 - 2 &= 7 - 2 \\ 5y &= 5 \\ \frac{5y}{5} &= \frac{5}{5} \\ y &= 1 \end{aligned}$$

24.
$$\begin{aligned} 5y + 1 &= 8y - 5 + 6y \\ 5y + 1 &= 8y + 6y - 5 \\ 5y + 1 &= 14y - 5 \\ 5y - 5y + 1 &= 14y - 5y - 5 \\ 1 &= 9y - 5 \\ 1 + 5 &= 9y - 5 + 5 \\ 6 &= 9y \\ \frac{6}{9} &= \frac{9y}{9} \\ \frac{2}{3} &= y \end{aligned}$$

25.
$$\begin{aligned} x - 3(2x + 3) &= 8 - 5x \\ x - 6x - 9 &= 8 - 5x \\ -5x - 9 &= 8 - 5x \\ -5x + 5x - 9 &= 8 - 5x + 5x \\ -9 &\neq 8 \end{aligned}$$

Because $-9 = 8$ is a contradiction, the equation has no solution.

26.
$$\begin{aligned} 9x - 10 &= 5x + 2(2x - 5) \\ 9x - 10 &= 5x + 4x - 10 \\ 9x - 10 &= 9x - 10 \end{aligned}$$

Because the equation is an identity, the solution is the set of all real numbers.

27.
$$\begin{aligned} 0.25x + 0.75(10 - x) &= 3 \\ 0.25x + 7.5 - 0.75x &= 3 \\ -0.50x + 7.5 &= 3 \\ -0.50x &= -4.5 \\ x &= 9 \end{aligned}$$

28.
$$\begin{aligned} 0.60x + 0.40(100 - x) &= 50 \\ 0.60x + 40 - 0.40x &= 50 \\ 0.20x &= 10 \\ x &= 50 \end{aligned}$$

29.
$$\begin{aligned} \frac{3x}{8} - \frac{4x}{3} &= 4 \\ (24)\frac{3x}{8} - (24)\frac{4x}{3} &= (24)4 \\ 9x - 32x &= 96 \\ -23x &= 96 \\ x &= -\frac{96}{23} \end{aligned}$$

30.
$$\begin{aligned} \frac{x}{5} - \frac{x}{2} &= 3 + \frac{3x}{10} \\ (10)\frac{x}{5} - (10)\frac{x}{2} &= (10)3 + (10)\frac{3x}{10} \\ 2x - 5x &= 30 + 3x \\ -6x &= 30 \\ x &= -5 \end{aligned}$$

31.
$$\begin{aligned} \frac{5x - 4}{5x + 4} &= \frac{2}{3} \\ 3(5x - 4) &= 2(5x + 4) \\ 15x - 12 &= 10x + 8 \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

32. $\dfrac{10x + 3}{5x + 6} = \dfrac{1}{2}$

$$\begin{aligned} 2(10x + 3) &= 1(5x + 6) \\ 20x + 6 &= 5x + 6 \\ 15x &= 0 \\ x &= 0 \end{aligned}$$

33. $10 - \dfrac{13}{x} = 4 + \dfrac{5}{x}$

$$\begin{aligned} \frac{10x - 13}{x} &= \frac{4x + 5}{x} \\ 10x - 13 &= 4x + 5 \\ 6x &= 18 \\ x &= 3 \end{aligned}$$

34. $\dfrac{15}{x} - 4 = \dfrac{6}{x} + 3$

$$\begin{aligned} \frac{15}{x} - \frac{6}{x} &= 7 \\ \frac{9}{x} &= 7 \\ 9 &= 7x \\ \frac{9}{7} &= x \end{aligned}$$

35. $3 = 2 + \dfrac{2}{z + 2}$

$$\begin{aligned} 3(z + 2) &= \left(2 + \frac{2}{z + 2}\right)(z + 2) \\ 3z + 6 &= 2z + 4 + 2 \\ z &= 0 \end{aligned}$$

36. $\dfrac{1}{x} + \dfrac{2}{x - 5} = 0$

$$\begin{aligned} x(x - 5)\frac{1}{x} + x(x - 5)\frac{2}{x - 5} &= x(x - 5)0 \\ x - 5 + 2x &= 0 \\ 3x - 5 &= 0 \\ 3x &= 5 \\ x &= \frac{5}{3} \end{aligned}$$

37. $\dfrac{x}{x + 4} + \dfrac{4}{x + 4} + 2 = 0$

$$\begin{aligned} \frac{x + 4}{x + 4} + 2 &= 0 \\ 1 + 2 &= 0 \\ 3 &\neq 0 \end{aligned}$$

Because $3 = 0$ is a contradiction, the equation has no solution.

38. $\dfrac{7}{2x + 1} - \dfrac{8x}{2x - 1} = -4$ Multiply each term by $(2x + 1)(2x - 1)$.

$$\begin{aligned} 7(2x - 1) - 8x(2x + 1) &= -4(2x + 1)(2x - 1) \\ 14x - 7 - 16x^2 - 8x &= -16x^2 + 4 \\ 6x &= 11 \\ x &= \frac{11}{6} \end{aligned}$$

39. $\dfrac{2}{(x - 4)(x - 2)} = \dfrac{1}{x - 4} + \dfrac{2}{x - 2}$ Multiply each term by $(x - 4)(x - 2)$.

$$\begin{aligned} 2 &= 1(x - 2) + 2(x - 4) \\ 2 &= x - 2 + 2x - 8 \\ 2 &= 3x - 10 \\ 12 &= 3x \\ 4 &= x \end{aligned}$$

A check reveals that $x = 4$ yields a denominator of zero. So, $x = 4$ is an extraneous solution, and the original equation has no real solution.

40. $\dfrac{12}{(x-1)(x+3)} = \dfrac{3}{x-1} + \dfrac{2}{x+3}$ Multiply each term by $(x-1)(x+3)$.

$$(x-1)(x+3)\frac{12}{(x-1)(x+3)} = (x-1)(x+3)\frac{3}{x-1} + (x-1)(x+3)\frac{2}{x+3}$$

$$\begin{aligned} 12 &= 3(x+3) + 2(x-1) \\ 12 &= 3x + 9 + 2x - 2 \\ 12 &= 5x + 7 \\ 5 &= 5x \\ x &= 1 \end{aligned}$$

A check reveals that $x = 1$ yields a denominator of zero. So, $x = 1$is an extraneous solution, and the original equation has no real solution.

41. $\dfrac{1}{x-3} + \dfrac{1}{x+3} = \dfrac{10}{x^2-9}$ Multiply each term by $(x+3)(x-3)$.

$$\begin{aligned} \frac{1}{x-3} + \frac{1}{x+3} &= \frac{10}{(x+3)(x-3)} \\ 1(x+3) + 1(x-3) &= 10 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

42. $\dfrac{1}{x-2} + \dfrac{3}{x+3} = \dfrac{4}{x^2+x-6}$

$\dfrac{1}{x-2} + \dfrac{3}{x+3} = \dfrac{4}{(x+3)(x-2)}$ Multiply each term by $(x+3)(x-2)$.

$$\begin{aligned} (x+3) + 3(x-2) &= 4 \\ x + 3 + 3x - 6 &= 4 \\ 4x - 3 &= 4 \\ 4x &= 7 \\ x &= \frac{7}{4} \end{aligned}$$

43. $$\begin{aligned} y &= 12 - 5x \\ 0 &= 12 - 5x \\ 5x &= 12 \\ x &= \tfrac{12}{5} \end{aligned} \qquad \begin{aligned} y &= 12 - 5x \\ y &= 12 - 5(0) \\ y &= 12 \end{aligned}$$

The x-intercept is $\left(\frac{12}{5}, 0\right)$ and the y-intercept is $(0, 12)$.

44. $$\begin{aligned} y &= 16 - 3x \\ 0 &= 16 - 3x \\ -16 &= -3x \\ \tfrac{16}{3} &= x \end{aligned} \qquad \begin{aligned} y &= 16 - 3x \\ y &= 16 - 3(0) \\ y &= 16 \end{aligned}$$

The x-intercept is $\left(\frac{16}{3}, 0\right)$ and the y-intercept is $(0, 16)$.

45. $$\begin{aligned} y &= -3(2x+1) \\ 0 &= -3(2x+1) \\ 0 &= 2x + 1 \\ x &= -\tfrac{1}{2} \end{aligned} \qquad \begin{aligned} y &= -3(2x+1) \\ y &= -3(2(0)+1) \\ y &= -3 \end{aligned}$$

The x-intercept is $\left(-\frac{1}{2}, 0\right)$ and the y-intercept is $(0, -3)$.

46. $$\begin{aligned} y &= 5 - (6 - x) \\ 0 &= 5 - (6 - x) \\ 0 &= -1 + x \\ 1 &= x \end{aligned} \qquad \begin{aligned} y &= 5 - (6 - x) \\ y &= 5 - (6 - 0) \\ y &= -1 \end{aligned}$$

The x-intercept is $(1, 0)$ and the y-intercept is $(0, -1)$.

47.
$$\begin{aligned} 2x + 3y &= 10 \\ 2x + 3(0) &= 10 \\ 2x &= 10 \\ x &= 5 \end{aligned} \qquad \begin{aligned} 2x + 3y &= 10 \\ 2(0) + 3y &= 10 \\ 3y &= 10 \\ y &= \tfrac{10}{3} \end{aligned}$$

The x-intercept is $(5, 0)$ and the y-intercept is $\left(0, \frac{10}{3}\right)$.

48.
$$\begin{aligned} 4x - 5y &= 12 \\ 4x - 5(0) &= 12 \\ 4x &= 12 \\ x &= 3 \end{aligned} \qquad \begin{aligned} 4x - 5y &= 12 \\ 4(0) - 5y &= 12 \\ -5y &= 12 \\ y &= -\tfrac{12}{5} \end{aligned}$$

The x-intercept is $(3, 0)$ and the y-intercept is $\left(0, -\frac{12}{5}\right)$.

49.
$$\begin{aligned} 4y - 0.75x + 1.2 &= 0 \\ 4(0) - 0.75x + 1.2 &= 0 \\ -0.75 + 1.2 &= 0 \\ x &= \frac{1.2}{0.75} = 1.6 \end{aligned} \qquad \begin{aligned} 4y - 0.75x + 1.2 &= 0 \\ 4y - 0.75(0) + 1.2 &= 0 \\ 4y + 1.2 &= 0 \\ y &= \frac{-1.2}{4} = -0.3 \end{aligned}$$

The x-intercept is $(1.6, 0)$ and the y-intercept is $(0, -0.3)$.

50.
$$\begin{aligned} 3y + 2.5x - 3.4 &= 0 \\ 3(0)2.5x - 3.4 &= 0 \\ 2.5x &= 3.4 \\ x &= \frac{3.4}{2.5} \\ x &= 1.36 \end{aligned} \qquad \begin{aligned} 3y + 2.5x - 3.4 &= 0 \\ 3y + 2.5(0) - 3.4 &= 0 \\ 3y &= 3.4 \\ y &= \frac{3.4}{3} \\ y &= 1.1\overline{3} \end{aligned}$$

The x-intercept is $(1.36, 0)$ and the y-intercept is $\left(0, 1.1\overline{3}\right)$.

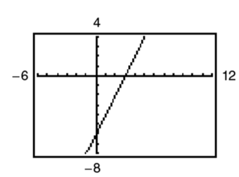
51.
$$\begin{aligned} \frac{2x}{5} + 8 - 3y &= 0 \\ 2x + 40 - 15y &= 0 \\ 2x + 40 - 15(0) &= 0 \\ 2x + 40 &= 0 \\ x &= -20 \end{aligned} \qquad \begin{aligned} 2x + 40 - 15y &= 0 \\ 2(0) + 40 - 15y &= 0 \\ 40 - 15y &= 0 \\ y &= \frac{40}{15} = \frac{8}{3} \end{aligned}$$

The x-intercept is $(-20, 0)$ and the y-intercept is $\left(0, \frac{8}{3}\right)$.

52.
$$\begin{aligned} \frac{8x}{3} + 5 - 2y &= 0 \\ \frac{8x}{3} + 5 - 2(0) &= 0 \\ \frac{8x}{3} &= -5 \\ x &= -\tfrac{15}{8} \end{aligned} \qquad \begin{aligned} \frac{8x}{3} + 5 - 2y &= 0 \\ \frac{8(0)}{3} + 5 - 2y &= 0 \\ 5 - 2y &= 0 \\ y &= \tfrac{5}{2} \end{aligned}$$

The x-intercept is $\left(-\frac{15}{8}, 0\right)$ and the y-intercept is $\left(0, \frac{5}{2}\right)$.

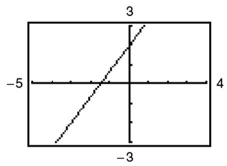
53. $y = 2(x - 1) - 4$



$$\begin{aligned} 0 &= 2(x - 1 - 4) \\ 0 &= 2x - 2 - 4 \\ 0 &= 2x - 6 \\ 6 &= 2x \\ 3 &= x \\ x &= 3 \end{aligned}$$

The x-intercept is $x = 3$. The solution of $0 = 2(x - 1) - 4$ and the x-intercept of $y = 2(x - 1) - 4$ are the same. They are both $x = 3$. The x-intercept is $(3, 0)$.

54. $y = \frac{4}{3}x + 2$

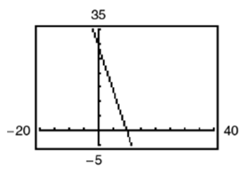


$$\begin{aligned} y &= \tfrac{4}{3}x + 2 \\ -\tfrac{4}{3}x &= 2 \\ \left(-\tfrac{3}{4}\right)\left(-\tfrac{4}{3}x\right) &= \left(-\tfrac{3}{4}\right)(2) \\ x &= -\tfrac{3}{2} \end{aligned}$$

Intercept: $\left(-\frac{3}{2}, 0\right)$

The solution to $0 = \frac{4}{3}x + 2$ is the same as the x-intercept of $y = \frac{4}{3}x + 2$. They are both $x = -\frac{3}{2}$.

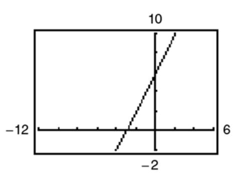
55. $y = 20 - (3x - 10)$



$$\begin{aligned} 0 &= 20 - (3x - 10) \\ 0 &= 20 - 3x + 10 \\ 0 &= 30 - 3x \\ 3x &= 30 \\ x &= 10 \end{aligned}$$

The x-intercept is $x = 10$. The solution of $0 = 20 - (3x - 10)$ and the x-intercept of $y = 20 - (3x - 10)$ are the same. They are both $x = 10$. The x-intercept is $(10, 0)$.

56. $y = 10 + 2(x - 2)$

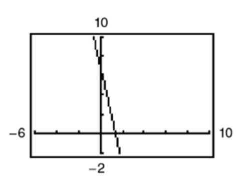


$$\begin{aligned} 0 &= 10 + 2(x - 2) \\ 0 &= 10 + 2x - 4 \\ 0 &= 6 + 2x \\ -2x &= 6 \\ x &= -3 \end{aligned}$$

Intercept: $(-3, 0)$

The solution to $0 = 10 + 2(x - 2)$ is the same as the x-intercept of $y = 10 + 2(x - 2)$. They are both $x = -3$.

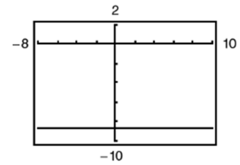
57. $y = -38 + 5(9 - x)$



$$\begin{aligned} 0 &= -38 + 5(9 - x) \\ 0 &= -38 + 45 - 5x \\ 0 &= 7 - 5x \\ 5x &= 7 \\ x &= \tfrac{7}{5} \end{aligned}$$

The x-intercept is at $x = \frac{7}{5}$. The solution of $0 = -38 + 5(9 - x)$ and the x-intercept of $y = -38 + 5(9 - x)$ are the same. They are both $x = \frac{7}{5}$. The x-intercept is $\left(\frac{7}{5}, 0\right)$.

58. $y = 6x - 6\left(\frac{16}{11} + x\right)$



$$\begin{aligned} 0 &= 6x - 6\left(\tfrac{16}{11} + x\right) \\ 0 &= 6x - \tfrac{96}{11} - 6x \\ 0 &\neq -\tfrac{96}{11} \end{aligned}$$

There is no x-intercept.

59. $\frac{2}{7.398} - \frac{4.405}{x} = \frac{1}{x}$ Multiply both sides by $7.398x$.

$$\begin{aligned} 2x - (4.405)(7.398) &= 7.398 \\ 2x &= (4.405)(7.398) + 7.398 \\ 2x &= (5.405)(7.398) \\ x &= \frac{(5.405)(7.398)}{2} \approx 19.993 \end{aligned}$$

60. $\frac{3}{6.350} - \frac{6}{x} = 18$ Multiply both sides by $6.350x$.

$$\begin{aligned} 3x - 6(6.350) &= 18(6.350)x \\ 3x - 38.1 &= 114.3x \\ -38.1 &= 111.3x \\ -0.342 &\approx x \end{aligned}$$

61.
$$\begin{aligned} 0.275x + 0.725(500 - x) &= 300 \\ 0.275x + 362.5 - 0.725x &= 300 \\ -0.45x &= -62.5 \\ x &= \frac{62.5}{0.45} \approx 138.889 \end{aligned}$$

62.
$$\begin{aligned} 2.763 - 4.5(2.1x - 5.1432) &= 6.32x + 5 \\ 2.763 - 9.45x + 23.1444 &= 6.32x + 5 \\ 20.9074 &= 15.77x \\ 1.326 &\approx x \end{aligned}$$

63.
$$\begin{aligned} 471 &= 2\pi(25) + 2\pi(5h) \\ 471 &= 50\pi + 10\pi h \\ 471 - 50\pi &= 10\pi h \\ h &= \frac{471 - 50\pi}{10\pi} = \frac{471 - 50(3.14)}{10(3.14)} = 10 \\ h &= 10 \text{ feet} \end{aligned}$$

64.
$$\begin{aligned} 248 &= 2(24) + 2(4x) + 2(6x) \\ 248 &= 48 + 8x + 12x \\ 200 &= 20x \\ x &= 10 \text{ centimeters} \end{aligned}$$

65. Let $y = 18$.

$$\begin{aligned} y &= 0.514x - 14.75 \\ 18 &= 0.514x - 14.75 \\ 32.75 &= 0.514x \\ \frac{32.75}{0.514} &= x \\ 63.7 &= x \end{aligned}$$

So, the height of the female is about 63.7 inches.

66. Let $y = 23$.

$$\begin{aligned} y &= 0.532x - 17.03 \\ 23 &= 0.532x - 17.03 \\ 40.03 &= 0.532x \\ \frac{40.03}{0.532} &= x \\ 75.2 &= x \end{aligned}$$

The height of the missing man is about 75.2 inches.

Because 75.2 in. $\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 6.27$ ft is about 6 feet 3 inches, it is possible the femur belongs to the missing man.

67. (a) The y-intercept is about $(0, 407)$.

(b) Let $t = 0$.

$$\begin{aligned} y &= 8.28t + 406.6 \\ &= 8.28(0) + 406.6 \\ &= 406.6 \end{aligned}$$

The y-intercept is $(0, 406.6)$.

In 2010, the population of Raleigh was about 406,600.

(c) Let $y = 448$.

$$\begin{aligned} 448 &= 8.28t + 406.6 \\ 41.4 &= 8.28t \\ \frac{41.4}{8.28} &= t \\ 5 &= t \end{aligned}$$

The population reached 448,000 in 2015.

68. (a) The y-intercept is approximately $(0, 102)$.

(b) Let $t = 0$.

$$\begin{aligned} y &= -0.77t + 101.8 \\ &= -0.77(0) + 101.8 \\ &= 101.8 \end{aligned}$$

The y-intercept is $(0, 101.8)$.

In 2010, the population of Flint was about 101,800.

(c) Let $y = 99.5$.

$$\begin{aligned} 99.5 &= -0.77t + 101.8 \\ -2.3 &= -0.77t \\ \frac{-2.3}{-0.77} &= t \\ 3 &\approx t \end{aligned}$$

The population was 99,500 in 2013.

69. Let $c = 10{,}000$.

$$\begin{aligned} c &= 0.37m + 2600 \\ 10{,}000 &= 0.37m + 2600 \\ 7400 &= 0.37m \\ \frac{7400}{0.37} &= m \\ m &= 20{,}000 \end{aligned}$$

So, the number of miles is 20,000.

70. Let $y = 1$.

$$\begin{aligned} y &= -0.25t + 8 \\ 1 &= -0.25t + 8 \\ 0.25t &= 7 \\ t &= 28 \text{ hours} \end{aligned}$$

71. $$\begin{aligned} x(3 - x) &= 10 \\ 3x - x^2 &= 10 \end{aligned}$$

False. This is a quadratic equation. The equation cannot be written in the form $ax + b = 0$.

72. $$\begin{aligned} 2(x + 3) &= 3x + 3 \\ 2x + 6 &= 3x + 3 \\ 3 &= x \end{aligned}$$

False, $x = 3$ is a solution.

73. $$\begin{aligned} 3(x - 1) - 2 &= 3x - 6 \\ 3x - 3 - 2 &= 3x - 6 \\ 3x - 5 &= 3x - 6 \\ -5 &\neq -6 \end{aligned}$$

False. The equation $-5 = -6$ is a contradiction, so the original equation has no solution.

74.
$$2 - \frac{1}{x-2} = \frac{3}{x-2}$$
$$(x-2)\left(2 - \frac{1}{x-2}\right) = (x-2)\left(\frac{3}{x-2}\right)$$
$$2(x-2) - 1 = 3$$
$$2x - 4 - 1 = 3$$
$$2x - 5 = 3$$
$$2x = 8$$
$$x = 4$$

False. $x = 4$ is a solution.

75. $\dfrac{3x+2}{5} = 7$

$3x + 2 = 35$ and $x + 9 = 20$

$3x = 33$ $\quad$ $x = 11$

$x = 11$

Yes, they are equivalent equations. They both have the solution $x = 11$.

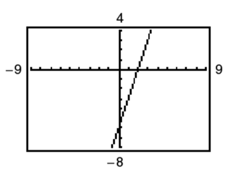
76. (a) The x-intercept is $(20{,}000, 0)$ and the y-intercept is $(0, 10{,}000)$. The subsidy is \$0 for an earned income of \$20,000. The subsidy is \$10,000 for an earned income of \$0.

(b) Set one of S or E equal to 0 and solve for the other.

(c) The earned income is \$8000.

(d) Set T equal to 14,000, substitute $10{,}000 - \frac{1}{2}E$ for S in the equation $T = E + S$, and solve for E.

77. (a)



(b) x-intercept: $(2, 0)$

(c) The x-intercept is the solution of the equation $3x - 6 = 0$.

78. (a) To find the x-intercept, let $y = 0$, and solve for x.
$$ax + by = c$$
$$ax + b(0) = c$$
$$ax = c$$
$$x = \frac{c}{a}$$

The x-intercept is $\left(\frac{c}{a}, 0\right)$.

(b) To find the y-intercept, let $x = 0$, and solve for y.
$$a(0) + by = c$$
$$by = c$$
$$y = \frac{c}{b}$$

The y-intercept is $\left(0, \frac{c}{b}\right)$.

(c) x-intercept:
$$x = \frac{c}{a} = \frac{11}{2}$$

The x-intercept is $\left(\frac{11}{2}, 0\right)$.

y-intercept:
$$y = \frac{c}{b} = \frac{11}{7}$$

The y-intercept is $\left(0, \frac{11}{7}\right)$.

79. Area of shaded region = Area of outer rectangle – Area of inner rectangle
$$A = 2x(2x+6) - x(x+4)$$
$$= 4x^2 + 12x - x^2 - 4x$$
$$= 3x^2 + 8x$$

80. Area of shaded region = Area of larger triangle – Area of smaller triangle
$$A = \tfrac{1}{2}(2x+8)^2 - \tfrac{1}{2}(x+4)^2$$
$$= \tfrac{1}{2}\left[4x^2 + 32x + 64 - \left(x^2 + 8x + 16\right)\right]$$
$$= \tfrac{1}{2}\left(3x^2 + 24x + 48\right)$$
$$= \tfrac{3}{2}x^2 + 12x + 24$$

81. $\frac{144}{x} = \frac{36}{55}$

$36x = 144(55)$

$x = \frac{7920}{36}$

$x = 220$

82. $\frac{19}{2} = \frac{x}{19}$

$2x = 19(19)$

$x = \frac{361}{2}$

83. $\frac{1}{9} = \frac{7}{x}$

$x = 7(9)$

$x = 63$

84. $\frac{x}{72} = \frac{18}{5}$

$5x = 18(72)$

$x = \frac{1296}{5}$

85. $\frac{x}{5} = \frac{5}{x}$

$x^2 = 25$

$x = \pm 5$

86. $\frac{14}{x} = \frac{15}{x+1}$

$14(x + 1) = 15x$

$14x + 14 = 15x$

$x = 14$

87. $(600)(0.24) = 144$

So, 144 is 24% of 600.

88. $0.55x = 110$

$x = \frac{110}{0.55}$

$x = 200$

So, 55% of 200 is 110.

89. $176x = 66.88$

$x = \frac{66.88}{176}$

$x = 0.38$

So, 38% of 176 is 66.88.

90. $(16x)y = 9x$

$y = \frac{9x}{16x}$

$y = \frac{9}{16} = 0.5625$

So, 56.25% of $16x$ is $9x$.

91. $\sqrt[3]{16x^5} = \left(2^4x^5\right)^{1/3} = 2x\sqrt[3]{2x^2}$

92. $\sqrt[5]{\frac{x^8z^4}{32}} = \frac{x}{2}\sqrt[5]{x^3z^4}$

93. $\sqrt[6]{x^3} = x^{3/6} = x^{1/2} = \sqrt{x}$

94. $\sqrt[6]{(x+1)^4} = (x+1)^{4/6} = (x+1)^{2/3}$

Section 1.3 Modeling with Linear Equations

1. mathematical modeling

2. verbal model; algebraic equation

3. A hidden equality is a statement that two algebraic expressions are equal.

4. Answers will vary. *Sample answers*:

Addition: sum, plus, increased by, more than, total of
Subtraction: difference, minus, less than, decreased by, subtracted from, reduced by

Multiplication: product, multiplied by, twice, times, percent of.

Division: quotient, divided by, ratio, per

Equality: equals, equal to, is, are, was, will be, represents

5. $y + 2$

The sum of a number and 2

A number increased by 2

6. $x - 8$

The difference of a number and 8

A number decreased by 8

7. $\frac{t}{6}$

A number divided by 6

8. $\frac{1}{3}u$

The product of $\frac{1}{3}$ and a number

9. $\frac{z - 2}{3}$

A number decreased by 2, then divided by 3

10. $\frac{x + 9}{5}$

A number increased by 9, then divided by 5

11. $-2(d + 5)$

The product of -2 and a number increased by 5

12. $10y(y - 3)$

The product of 10, a number, and 3 less than the same number

13. *Verbal Model:* Product $=$ (first odd integer) $\cdot$ (second odd integer)

Labels: Product $= P$, first odd integer $= 2n - 1$, second odd integer $= 2n - 1 + 2 = 2n + 1$

Equation: $P = (2n - 1)(2n + 1) = 4n^2 - 1$

14. *Verbal Model:* (Sum) $=$ (first even number)^2 $+$ (second even number)^2

Labels: Sum $= S$, first even number $= 2n$, second even number $= 2n + 2$

Equation: $S = (2n)^2 + (2n + 2)^2 = 4n^2 + 4n^2 + 8n + 4 = 8n^2 + 8n + 4$

15. *Verbal Model:* (Distance) $=$ (rate) $\cdot$ (time)

Labels: Distance $= d$, rate $= 55$ mph, time $= t$

Equation: $d = 55t$

16. *Verbal Model:* (time) $=$ (distance) $\div$ (rate)

Labels: time $= t$, distance $= 900$ km, rate $= r$

Equation: $t = \frac{900}{r}$

17. *Verbal Model:* (Amount of acid) $= 20\% \cdot$ (amount of solution)

Labels: Amount of acid (in gallons) $= A$, amount of solution (in gallons) $= x$

Equation: $A = 0.20x$

18. *Verbal Model:* (Sale price) $=$ (list price) $-$ (discount)

Labels: Sale price $= S$, list price $= L$, discount $= 0.33L$

Equation: $S = L - 0.33L = 0.67L$

19. *Verbal Model:* Perimeter $= 2$(width) $+ 2$(length)

Labels: Perimeter $= P$, width $= x$, length $= 2$(width) $= 2x$

Equation: $P = 2x + 2(2x) = 6x$

20. *Verbal Model:* (Area) $= \frac{1}{2}$(base)(height)

Labels: Area $= A$, base $= 16$ in., height $= h$

Equation: $A = \frac{1}{2}(16)h = 8h$

21. *Verbal Model:* $(\text{Total cost}) = (\text{unit cost})(\text{number of units}) + (\text{fixed cost})$

Labels: Total cost $= C$, unit cost $= \$40$, number of units $= x$, fixed cost $= \$2500$

Equation: $C = 2500 + 40x$

22. *Verbal Model:* $(\text{Revenue}) - (\text{price})(\text{number of units})$

Labels: Revenue $= R$, price $= \$12.99$, number of units $= x$

Equation: $R = 12.99x$

23. *Verbal Model:* $(\text{Discount}) = (\text{percent}) \cdot (\text{list price})$

Equation: $d = 0.30L$

24. *Labels:* A = amount of water, q = number of quarts

Verbal Model: $(\text{Amount of water}) = \dfrac{(\text{percent})}{100} \cdot (\text{number of quarts})$

Equation: $A = 0.72q$

25. *Labels:* N = the number, p = percent % of the number

Verbal Model: $(\text{The number}) = \dfrac{(\text{percent})}{100} \cdot 672$

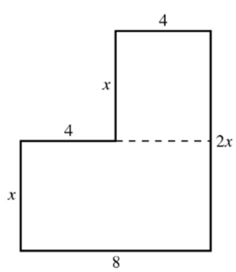
Equation: $N = \dfrac{p}{100} \cdot 672$

26. *Labels:* S_2 = sales for this month, S_1 = sales from last month

Verbal Model: $(\text{Sales for this month}) = (\text{Sales from last month}) + \dfrac{(\text{percent})}{100} \cdot (\text{Sales from last month})$

Equation: $S_2 = S_1 + 0.2S_1$

27.



Area = Area of top rectangle + Area of bottom rectangle

$A = 4x + 8x = 12x$

28. Area $= \frac{1}{2}(\text{base})(\text{height})$

$$A = \frac{1}{2}\left(\frac{2}{3}b + 1\right) = \frac{1}{3}b^2 + \frac{1}{2}b$$

29. *Verbal Model:* $\text{Sum} = (\text{first number}) + (\text{second number})$

Labels: Sum $= 525$, first number $= n$, second number $= n + 1$

Equation:

$525 = n + (n + 1)$

$525 = 2n + 1$

$524 = 2n$

$n = 262$

Answer: First number $= n = 262$, second number $= n + 1 = 263$

30. *Verbal Model:* Sum = (first number) + (second number) + (third number)

Labels: Sum = 804, first number = n, second number = $n + 1$, third number = $n + 2$

Equation:

$$804 = n + n + 1 + n + 2$$
$$804 = 3n + 3$$
$$801 = 3n$$
$$267 = n$$

Answer: $n = 267$, $n + 1 = 268$ (second number), and $n + 2 = 269$ (third number)

31. *Verbal Model:* Difference = (one number) − (another number)

Labels: Difference = 148, one number = $5x$, another number = x

Equation:

$$148 = 5x - x$$
$$148 = 4x$$
$$x = 37$$
$$5x = 185$$

Answer: The two numbers are 37 and 185.

32. *Verbal Model:* Difference = (number) − (one-fifth of number)

Labels: Difference = 76, number = n, one-fifth of number = $\frac{1}{5}n$

Equation:

$$76 = n - \tfrac{1}{5}n$$
$$76 = \tfrac{4}{5}n$$
$$95 = n$$

Answer: The numbers are 95 and $\frac{1}{5} \cdot 95 = 19$.

33. *Verbal Model:* Product = (smaller number) · (larger number) = $(\text{smaller number})^2 - 5$

Labels: Smaller number = n, larger number = $n + 1$

Equation:

$$n(n + 1) = n^2 - 5$$
$$n^2 + n = n^2 - 5$$
$$n = -5$$

Answer: Smaller number = $n = -5$, larger number = $n + 1 = -4$

34. *Verbal Model:* Difference = (reciprocal of smaller number) − (reciprocal of larger number)

$= \frac{1}{4}$ (reciprocal of smaller number)

Labels: Smaller number = n, larger number = $n + 1$, difference = $\frac{1}{4n}$

Equation:

$$\frac{1}{4n} = \frac{1}{n} - \frac{1}{n + 1} \qquad \text{Multiply both sides by } 4n(n + 1).$$
$$4n(n + 1)\frac{1}{4n} = 4n(n + 1)\frac{1}{n} - 4n(n + 1)\frac{1}{n + 1}$$
$$n + 1 = 4(n + 1) - 4n$$
$$n + 1 = 4n + 4 - 4n$$
$$n = 3$$

Answer: The numbers are 3 and $n + 1 = 4$.

35. *Verbal Model:* (first paycheck) + (second paycheck) = total

Labels: second paycheck $= x$, first paycheck $= 0.85x$, total = \$1125

Equation:

$$\begin{aligned} 0.85x + x &= 1125 \\ 1.85x &= 1125 \\ x &\approx 608.11 \\ 0.85x &\approx 516.89 \end{aligned}$$

Answer: The first salesperson's weekly paycheck is \$516.89 and the second salesperson's weekly paycheck is \$608.11.

36. Let P be the price of the ticket.

$$\begin{aligned} P(1 - 0.165) &= 116.90 \\ P(0.835) &= 116.90 \\ P &= \frac{116.90}{0.835} = 140 \end{aligned}$$

The original list price is \$140.00.

37. *Verbal Model:* (Loan payments) = (Percent) · (Annual Income)

Labels: Loan payments = 15,680 (dollars)

Percent = 0.32

Annual income = I (dollars)

Equation:

$$\begin{aligned} 15{,}680 &= 0.32I \\ \frac{15{,}680}{0.32} &= \frac{0.32I}{0.32} \\ 49{,}000 &= I \end{aligned}$$

Answer: The family's annual income is \$49,000.

38. *Verbal Model:* (Mortgage payment) = (Percent) · (Monthly income)

Labels: Mortgage payment = 760 (dollars)

Percent = 0.16

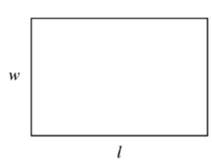
Monthly income = I (dollars)

Equation:

$$\begin{aligned} 760 &= 0.16I \\ \frac{760}{0.16} &= \frac{0.16I}{0.16} \\ 4750 &= I \end{aligned}$$

Answer: The family's monthly income is \$4750.

39. (a)



(b)

$$\begin{aligned} l &= 1.5w \\ P &= 2l + 2w \\ &= 2(1.5w) + 2w \\ &= 5w \end{aligned}$$

(c)

$$\begin{aligned} 25 &= 5w \\ 5 &= w \end{aligned}$$

Width: $w = 5$ meters

Length: $l = 1.5w = 7.5$ meters

Dimensions: 7.5 meters × 5 meters

40. Let x be the length of the field. The width is $\frac{2}{3}x$, and the perimeter is 400.

$$2x + 2\left(\tfrac{2}{3}\right)x = 400$$
$$\frac{6}{3}x + \frac{4}{3}x = 400$$
$$\frac{10}{3}x = 400$$
$$x = \frac{3(400)}{10} = 120$$

The dimensions are 120 yards by 80 yards.

41. *Verbal Model:* $\text{Average} = \dfrac{(\text{test \#1}) + (\text{test \#2}) + (\text{test \#3}) + (\text{test \#4})}{4}$

Labels: Average = 90, test #1 = 87, test #2 = 92, test #3 = 84, test #4 = x

Equation: $90 = \dfrac{87 + 92 + 84 + x}{4}$

Answer: You must score 97 or better on test #4 to earn an A for the course.

42. *Verbal Model:* $\text{Average} = \dfrac{(\text{test \#1}) + (\text{test \#2}) + (\text{test \#3}) + (\text{test \#4})}{5}$

Labels: Average = 90, test #1 = 87, test #2 = 92, test #3 = 84, test #4 = x

Equation:
$$90 = \frac{87 + 92 + 84 + x}{5}$$
$$450 = 87 + 92 + 84 + x$$
$$450 = 263 + x$$
$$187 = x$$

Answer: You must score 187 out of 200 on the last test to get an A in the course.

43. $\text{Rate} = \dfrac{\text{distance}}{\text{time}} = \dfrac{50 \text{ kilometers}}{\frac{1}{2} \text{ hour}} = 100 \text{ kilometers/hour}$

$\text{Total time} = \dfrac{\text{total distance}}{\text{rate}} = \dfrac{500 \text{ kilometers}}{100 \text{ kilometers/hour}} = 5 \text{ hours}$

The entire trip takes 5 hours.

44. *Verbal Model:* $(\text{Distance}) = (\text{rate})(\text{time}_1 + \text{time}_2)$

Labels: Distance $= 2 \cdot 200 = 400$ miles, rate = 2,

$\text{time}_1 = \dfrac{\text{distance}}{\text{rate}_1} = \dfrac{200}{55}$ hours,

$\text{time}_2 = \dfrac{\text{distance}}{\text{rate}_2} = \dfrac{200}{40}$ hours

Equation:
$$400 = r\left(\frac{200}{55} + \frac{200}{40}\right)$$
$$400 = r\left(\frac{1600}{440} + \frac{2200}{440}\right) = \frac{3800}{440}r$$
$$46.3 \approx r$$

The average speed for the round trip was approximately 46.3 miles per hour.

45. *Verbal Model:* $(\text{Distance}) = (\text{rate})(\text{time})$

Labels: Distance $= 1.5 \times 10^{11}$ (meters)

Rate $= 3.0 \times 10^{8}$ (meters per second)

Time $= t$

Equation: $1.5 \times 10^{11} = (3.0 \times 10^{8})t$

$500 = t$

Light from the sun travels to the Earth in 500 seconds or approximately 8.33 minutes.

46. *Verbal Model:* $\text{time} = \dfrac{\text{distance}}{\text{rate}}$

Equation: $t = \dfrac{3.84 \times 10^{8} \text{ meters}}{3.0 \times 10^{8} \text{ meters per second}}$

$t = 1.28$ seconds

The radio wave travels from Mission Control to the moon in 1.28 seconds.

47. *Verbal Model:* $\dfrac{(\text{Height of building})}{(\text{Length of building's shadow})} = \dfrac{(\text{Height of post})}{(\text{Length of post's shadow})}$

Labels: Height of building $= x$ (feet)

Length of building's shadow $= 105$ (feet)

Height of post $= 3 \cdot 12 = 36$ (inches)

Length of post's shadow $= 4$ (inches)

Equation: $\dfrac{x}{105} = \dfrac{36}{4}$

$x = 945$

One Liberty Place is 945 feet tall.

48. *Verbal Model:* $\dfrac{(\text{height of tree})}{(\text{length of tree's shadow})} = \dfrac{(\text{height of lamppost})}{(\text{length of lamppost's shadow})}$

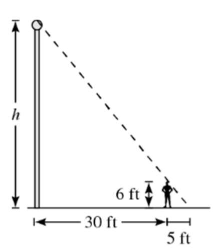
Labels: height of tree $= h$, height of tree's shadow $= 8$ meters,
height of lamppost $= 2$ meters, height of lamppost's shadow $= 0.75$ meter

Equation: $\dfrac{h}{8} = \dfrac{2}{0.75}$

$h = \dfrac{8(2)}{0.75} = 21\frac{1}{3}$

The tree is $21\frac{1}{3}$ meters tall.

49. (a)



(b) *Verbal Model:* $\dfrac{(\text{height of pole})}{(\text{height of pole's shadow})} = \dfrac{(\text{height of person})}{(\text{height of person's shadow})}$

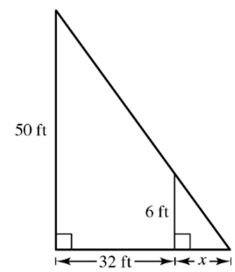
Labels: Height of pole $= h$, height of pole's shadow $= 30 + 5 = 35$ feet,
height of person $= 6$ feet, height of person's shadow $= 5$ feet

Equation: $\dfrac{h}{35} = \dfrac{6}{5}$

$h = \dfrac{6}{5} \cdot 35 = 42$

The pole is 42 feet tall.

50. *Verbal Model:* $\dfrac{(\text{height of tower})}{(\text{height of tower's shadow})} = \dfrac{(\text{height of person})}{(\text{height of person's shadow})}$



Labels: Let x = length of person's shadow.

$$\frac{50}{32 + x} = \frac{6}{x}$$

$$50x = 6(32 + x)$$

Equation:

$$50x = 192 + 6x$$

$$44x = 192$$

$$x \approx 4.36 \text{ feet}$$

51. *Verbal Model:* $\boxed{\text{Interest from } 4\tfrac{1}{2}\%} + \boxed{\text{Interest from } 5\%} = \boxed{\text{Total interest}}$

Labels: Amount invested at $4\frac{1}{2}\% = x$ dollars

Amount invested at $5\% = 12{,}000 - x$ dollars

Interest from $4\frac{1}{2}\% = x(0.045)$ dollars

Interest from $5\% = (12{,}000 - x)(0.05)$ dollars

Total annual interest $= 580$ dollars

Equation:

$$0.045x + 0.05(12{,}000 - x) = 580$$

$$0.045x + 600 - 0.05x = 580$$

$$-0.005x = -20$$

$$x = 4000$$

So, \$4000 was invested at $4\frac{1}{2}\%$ and $\$12{,}000 - \$4000 = \$8000$ was invested at 5%.

52. *Verbal Model:* $\boxed{\text{Interest from } 3\%} + \boxed{\text{Interest from } 4\tfrac{1}{2}\%} = \boxed{\text{Total interest}}$

Labels: Amount invested at $3\% = x$

Amount invested at $4\frac{1}{2}\% = 25{,}000 - x$

Equation:

$$0.03x + 0.045(25{,}000 - x) = 900$$

$$0.03x + 1125 - 0.045x = 900$$

$$-0.015x = -225$$

$$x = 15{,}000$$

So, \$15,000 was invested at 3% and $\$25{,}000 - \$15{,}000 = \$10{,}000$ was invested at $4\frac{1}{2}\%$.

53. *Verbal Model:* (Profit from dogwood trees) + (profit from red maple trees) = (total profit)

Labels: Inventory of dogwood trees $= x$, inventory of red maple trees $= 40{,}000 - x$, profit from dogwood trees $= 0.25x$, profit from red maple trees $= 0.17(40{,}000 - x)$, total profit $= 0.20(40{,}000) = 8000$

Equation:

$$0.25x + 0.17(40{,}000) = 8000$$

$$0.25x + 6800 - 0.17x = 8000$$

$$0.08x = 1200$$

$$x = 15{,}000$$

The amount invested in dogwood trees was \$15,000 and the amount invested in red maple trees was $\$40{,}000 - \$15{,}000 = \$25{,}000$.

54. *Verbal Model:* (Profit from all-electric) + (profit from hybrid vehicles) = (total profit)

Labels: Inventory of all-electric $= x$, inventory of hybrid vehicles $= 600{,}000 - x$,
profit from all-electric $= 0.24x$, profit from hybrid vehicles $= 0.28(600{,}000 - x)$,
total profit $= 0.25(600{,}000) = 150{,}000$

Equation:

$$0.24x + 0.28(600{,}000 - x) = 150{,}000$$
$$0.24x + 168{,}000 - 0.28x = 150{,}000$$
$$-0.04x = -18{,}000$$
$$x = 450{,}000$$

The amount invested in all-electric was \$450,000 and the amount invested in hybrid vehicles was \$600,000 − \$450,000 = \$150,000.

55. *Verbal Model:* [Amount of gasoline in mixture] + [Amount of gasoline to add] = [Amount of gasoline in final mixture]

Labels: Amount of gasoline in mixture $= \frac{32}{33}(2)$ (gallons)
Amount of gasoline to add $= x$ (gallons)
Amount of gasoline in final mixture $= \frac{50}{51}(2 + x)$ (gallons)

Equation:

$$\frac{64}{33} + x = \frac{50}{51}(2 + x)$$
$$\frac{64}{33} + x = \frac{100}{51} + \frac{50}{51}x$$
$$3264 + 1683x = 3300 + 1650x$$
$$33x = 36$$
$$x \approx 1.09$$

The forester should add about 1.09 gallons of gasoline to the mixture.

56. *Verbal Model:* $\begin{pmatrix}\text{Price per pound}\\\text{of peanuts}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\\text{peanuts}\end{pmatrix} + \begin{pmatrix}\text{price per pound}\\\text{of walnuts}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\\text{walnuts}\end{pmatrix} = \begin{pmatrix}\text{price per pound}\\\text{of nut mixture}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\\text{nut mixture}\end{pmatrix}$

Labels: Price per pound of peanuts = \$1.49, pounds of peanuts $= x$, price per pound of peanuts = \$2.69,
pounds of walnuts $= 100 - x$, price per pound of nut mixture = \$2.21, pounds of nut mixture = 100

Equation:

$$1.49x + 2.69(100 - x) = 2.21(100)$$
$$1.49x + 2.69 - 2.69x = 2.21$$
$$-1.2x = -48$$
$$x = 40$$

There were 40 pounds of peanuts and $100 - 40 = 60$ pounds of walnuts in the mixture.

57.

$$A = \frac{1}{2}bh$$
$$2A = bh$$
$$\frac{2A}{b} = h$$

58.

$$V = lwh$$
$$\frac{V}{wh} = l$$

59.

$$S = C + RC$$
$$S = C(1 + R)$$
$$\frac{S}{1 + R} = C$$

60.

$$S = L - RL$$
$$S = L(1 - R)$$
$$\frac{S}{1 - R} = L$$

61.
$$A = P + Prt$$
$$A - P = Prt$$
$$\frac{A - P}{Pt} = r$$

62.
$$A = \frac{1}{2}(a + b)h$$
$$2A = (a + b)h$$
$$\frac{2A}{h} = a + b$$
$$b = \frac{2A}{h} - a$$

63.
$$C = \tfrac{5}{9}(F - 32)$$
$$= \tfrac{5}{9}(98.6 - 32)$$
$$= \tfrac{5}{9}(66.6)$$
$$= 37°$$

The temperature is $37°C$.

64.
$$F = \tfrac{9}{5}C + 32$$
$$= \tfrac{9}{5}(27) + 32$$
$$= 48.6 + 32$$
$$= 80.6°F$$

The temperature is $80.6°F$.

65.
$$V = \frac{4}{3}\pi r^3$$
$$5.96 = \frac{4}{3}\pi r^3$$
$$17.88 = 4\pi r^3$$
$$\frac{17.88}{4\pi} = r^3$$
$$r = \sqrt[3]{\frac{4.47}{\pi}} \approx 1.12 \text{ inches}$$

66.
$$V = \pi r^2 h$$
$$h = \frac{V}{\pi r^2} = \frac{603.2}{\pi(2)^2} \approx 48 \text{ feet}$$

67. True. The expression $\frac{x^3}{(x - 4)^2}$ can be described as x cubed divided by the square of the difference of x and 4.

68. Area of circle: $A = \pi r^2 = \pi(2)^2 = 4\pi \approx 12.56 \text{ in.}^2$

Area of square: $A = s^2 = (4)^2 = 16 \text{ in.}^2$

True. $12.56 \text{ in.}^2 < 16 \text{ in.}^2$, so the area of the circle is less than the area of the square.

69. One possible interpretation: $\frac{5}{3n}$

Another possible interpretation: $\frac{5}{n} \cdot 3 = \frac{15}{n}$

The phrase "the quotient of 5 and a number" indicates the variable is in the denominator.

So, the expression $\frac{3n}{5}$ is not a possible interpretation.

70. (a) *Verbal Model:* $\frac{\text{Height of building}}{\text{Length of building's shadow}} = \frac{\text{Height of post}}{\text{Length of post's shadow}}$

(b) *Equation:* $\frac{x}{30} = \frac{4}{3}$

71. $d = rt \Rightarrow t = \frac{d}{r}$, not $t = \frac{r}{d}$

72. $P = 2l + 2w \Rightarrow w = \frac{(P - 2l)}{2} = \frac{P}{2 - l}$, not $P - l$

73. $(x + \sqrt{3})(x - \sqrt{3}) = x^2 - x\sqrt{3} + x\sqrt{3} - 3 = x^2 - 3$

74. $(x + 3\sqrt{2})(x - 3\sqrt{2}) = x^2 - (3\sqrt{2})^2 = x^2 - 18$

75. $\left(x - 3 + \sqrt{7}\right)\left(x - 3 - \sqrt{7}\right) = (x - 3)^2 - \left(\sqrt{7}\right)^2$
$= x^2 - 6x + 9 - 7$
$= x^2 - 6x + 2$

76. $\left(x + \sqrt{2} + 2\right)\left(x + \sqrt{2} - 2\right) = \left(x + \sqrt{2}\right)^2 - 4$
$= x^2 + 2\sqrt{2}x + 2 - 4$
$= x^2 + 2\sqrt{2}x - 2$

77. $4x^2 + 4x + 1 = (2x + 1)^2$

78. $x^2 - 22x + 121 = (x - 11)(x - 11) = (x - 11)^2$

79. $u^3 + 27v^3 = (u + 3v)\left(u^2 - 3uv + 9v^2\right)$

80. $(x + 2)^3 - y^3 = (x + 2 - y)\left[(x + 2)^2 + (x + 2)y + y^2\right]$

81. $2x^2 + 9x + 4 = (2x + 1)(x + 4)$

82. $2x^2 - 3x - 5 = (x + 1)(2x - 5)$

83. $\dfrac{-3 + \sqrt{3^2 - 4(-9)}}{2} = \dfrac{-3 + \sqrt{45}}{2} = \dfrac{-3 + 3\sqrt{5}}{2}$

84. $\dfrac{-2 - \sqrt{2^2 - 4(3)(-10)}}{2(3)} = \dfrac{-2 - \sqrt{124}}{6}$
$= \dfrac{-2 - 2\sqrt{31}}{6}$
$= \dfrac{-1 - \sqrt{31}}{3}$

85. $\left[3\left(2 - \sqrt{2}\right) - 6\right]^2 - 18 = \left[3(2) - 3\sqrt{2} - 6\right]^2 - 18$
$= \left(-3\sqrt{2}\right)^2 - 18$
$= 9(2) - 18$
$= 0$

86. $\left(-1 + \sqrt{7}\right)^2 + 2\left(-1 + \sqrt{7}\right) - 6 = \left(1 - 2\sqrt{7} + 7\right) - 2 + 2\sqrt{7} - 6 = 0$

87. $9.46 \times 10^{12} = 9{,}460{,}000{,}000{,}000$

88. $9.02 \times 10^{-6} = 0.00000902$

89. $-3.75 \times 10^{-4} = -0.000375$

90. $1.83 \times 10^{8} = 183{,}000{,}000$

Section 1.4 Quadratic Equations and Applications

1. quadratic equation

2. second-degree polynomial

3. discriminant

4. Pythagorean Theorem

5. Four methods to solve a quadratic equation are: factoring, extracting square roots, completing the square, and using the Qualitative Formula.

6. The height of an object that is falling is given by the equation $s = -16t^2 + v_0t + s_0$, where s is the height, v_0 is the initial velocity, and s_0 is the initial height.

7.
$$6x^2 + 3x = 0$$
$$3x(2x + 1) = 0$$
$$3x = 0 \quad \text{or} \quad 2x + 1 = 0$$
$$x = 0 \quad \text{or} \quad x = -\tfrac{1}{2}$$

8.
$$8x^2 - 2x = 0$$
$$2x(4x - 1) = 0$$
$$8x = 0 \quad \text{or} \quad 4x - 1 = 0$$
$$x = 0 \quad \text{or} \quad x = \tfrac{1}{4}$$

9.
$$3 + 5x - 2x^2 = 0$$
$$(3 - x)(1 + 2x) = 0$$
$$3 - x = 0 \quad \text{or} \quad 1 + 2x = 0$$
$$x = 3 \quad \text{or} \quad x = -\tfrac{1}{2}$$

10.
$$x^2 + 6x + 9 = 0$$
$$(x + 3)(x + 3) = 0$$
$$x + 3 = 0$$
$$x = -3$$

11.
$$x^2 + 10x + 25 = 0$$
$$(x + 5)(x + 5) = 0$$
$$x + 5 = 0$$
$$x = -5$$

12.
$$4x^2 + 12x + 9 = 0$$
$$(2x + 3)(2x + 3) = 0$$
$$2x + 3 = 0 \Rightarrow x = -\tfrac{3}{2}$$

13.
$$16x^2 - 9 = 0$$
$$(4x + 3)(4x + 3) = 0$$
$$4x + 3 = 0 \Rightarrow x = -\tfrac{3}{4}$$
$$4x - 3 = 0 \Rightarrow x = \tfrac{3}{4}$$

14.
$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$
$$x - 4 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 4 \quad \text{or} \quad x = -2$$

15.
$$2x^2 = 19x + 33$$
$$2x^2 - 19x - 33 = 0$$
$$(2x + 3)(x - 11) = 0$$
$$2x + 3 = 0 \Rightarrow x = -\tfrac{3}{2}$$
$$x - 11 = 0 \Rightarrow x = 11$$

16.
$$-x^2 + 4x = 3$$
$$-x^2 + 4x - 3 = 0$$
$$(-1)(-x^2 + 4x - 3) = (-1)(0)$$
$$x^2 - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x - 1 = 0 \Rightarrow x = 1$$

17.
$$\tfrac{3}{4}x^2 + 8x + 20 = 0$$
$$4\left(\tfrac{3}{4}x^2 + 8x + 20\right) = 4(0)$$
$$3x^2 + 32x + 80 = 0$$
$$(3x + 20)(x + 4) = 0$$
$$3x + 20 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = -\tfrac{20}{3} \quad \text{or} \quad x = -4$$

18.
$$\tfrac{1}{8}x^2 - x - 16 = 0$$
$$x^2 - 8x - 128 = 0$$
$$(x - 16)(x + 8) = 0$$
$$x - 16 = 0 \Rightarrow x = 16$$
$$x + 8 = 0 \Rightarrow x = -8$$

19.
$$x^2 = 49$$
$$x = \pm 7$$

20.
$$x^2 = 144$$
$$x = \pm 12$$

21. $x^2 = 19$

$x = \pm\sqrt{19}$

$x \approx \pm 4.36$

22. $x^2 = 43$

$x = \pm\sqrt{43}$

$x \approx 6.56$

23. $3x^2 = 81$

$x^2 = 27$

$x = \pm 3\sqrt{3}$

$\approx \pm 5.20$

24. $9x^2 = 36$

$x^2 = 4$

$x = \pm\sqrt{4} = \pm 2$

25. $(x - 4)^2 = 49$

$x - 4 = \pm 7$

$x = 4 \pm 7$

$x = 11$ or $x = -3$

26. $(x - 5)^2 = 25$

$x - 5 = \pm 5$

$x = 5 \pm 5$

$x = 0$ or $x = 10$

27. $(x + 2)^2 = 14$

$x + 2 = \pm\sqrt{14}$

$x = -2 \pm \sqrt{14}$

$\approx 1.74, \ -5.74$

28. $(x + 9)^2 = 24$

$x + 9 = \pm\sqrt{24}$

$x = -9 \pm 2\sqrt{6}$

$\approx -4.10, \ -13.90$

29. $(2x - 1)^2 = 18$

$2x - 1 = \pm\sqrt{18}$

$2x = 1 \pm 3\sqrt{2}$

$x = \dfrac{1 \pm 3\sqrt{2}}{2}$

$\approx 2.62, \ -1.62$

30. $(4x + 7)^2 = 44$

$4x + 7 = \pm\sqrt{44}$

$4x = -7 \pm 2\sqrt{11}$

$x = \dfrac{-7 \pm 2\sqrt{11}}{4} = -\dfrac{7}{4} \pm \dfrac{\sqrt{11}}{2}$

$\approx -0.09, \ -3.41$

31. $(x - 7)^2 = (x + 3)^2$

$x - 7 = \pm(x + 3)$

$x - 7 = x + 3$ or $x - 7 = -x - 3$

$-7 \neq 3$ or $2x = 4$

$x = 2$

The only solution of the equation is $x = 2$.

32. $(x + 5)^2 = (x + 4)^2$

$x + 5 = \pm(x + 4)$

$x + 5 = +(x + 4)$ or $x + 5 = -(x + 4)$

$5 \neq 4$ or $x + 5 = -x - 4$

$2x = -9$

$x = -\frac{9}{2}$

The only solution of the equation is $x = -\frac{9}{2}$.

33. $x^2 + 4x - 32 = 0$

$x^2 + 4x = 32$

$x^2 + 4x + 2^2 = 32 + 2^2$

$(x + 2)^2 = 36$

$x + 2 = \pm 6$

$x = -2 \pm 6$

$x = 4$ or $x = -8$

34. $x^2 - 2x - 3 = 0$

$x^2 - 2x = 3$

$x^2 - 2x + (-1)^2 = 3 + (1)^2$

$(x - 1)^2 = 4$

$x - 1 = \pm\sqrt{4}$

$x = 1 \pm 2$

$x = 3$ or $x = -1$

35.
$$\begin{aligned} x^2 + 4x + 2 &= 0 \\ x^2 + 4x &= -2 \\ x^2 + 4x + 2^2 &= -2 + 2^2 \\ (x+2)^2 &= 2 \\ x + 2 &= \pm\sqrt{2} \\ x &= -2 \pm \sqrt{2} \end{aligned}$$

36.
$$\begin{aligned} x^2 + 8x + 14 &= 0 \\ x^2 + 8x &= -14 \\ x^2 + 8x + 4^2 &= -14 + 16 \\ (x+4)^2 &= 2 \\ x + 4 &= \pm\sqrt{2} \\ x &= -4 \pm \sqrt{2} \end{aligned}$$

37.
$$\begin{aligned} 6x^2 - 12x &= -3 \\ x^2 - 2x &= -\frac{1}{2} \\ x^2 - 2x + 1^2 &= -\frac{1}{2} + 1^2 \\ (x-1)^2 &= \frac{1}{2} \\ x - 1 &= \pm\sqrt{\frac{1}{2}} \\ x &= 1 \pm \sqrt{\frac{1}{2}} \\ x &= 1 \pm \frac{\sqrt{2}}{2} \end{aligned}$$

38.
$$\begin{aligned} 4x^2 - 4x &= 1 \\ x^2 - x &= \frac{1}{4} \\ x^2 - x + \left(-\frac{1}{2}\right)^2 &= \frac{1}{4} + \left(-\frac{1}{2}\right)^2 \\ \left(x - \frac{1}{2}\right)^2 &= \frac{1}{2} \\ x - \frac{1}{2} &= \pm\frac{\sqrt{2}}{2} \\ x &= \frac{1}{2} \pm \frac{\sqrt{2}}{2} \\ x &= \frac{1 \pm \sqrt{2}}{2} \end{aligned}$$

39.
$$\begin{aligned} 7 + 2x - x^2 &= 0 \\ -x^2 + 2x + 7 &= 0 \\ x^2 - 2x - 7 &= 0 \\ x^2 - 2x &= 7 \\ x^2 - 2x + (-1)^2 &= 7 + (-1)^2 \\ (x-1)^2 &= 8 \\ x - 1 &= \pm 2\sqrt{2} \\ x &= 1 \pm 2\sqrt{2} \end{aligned}$$

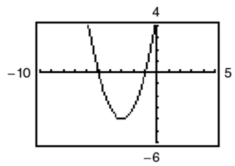
40.
$$\begin{aligned} -x^2 + x - 1 &= 0 \\ x^2 - x + 1 &= 0 \\ x^2 - x + \tfrac{1}{4} &= -1 + \tfrac{1}{4} \\ \left(x - \tfrac{1}{2}\right)^2 &= -\tfrac{3}{4} \end{aligned}$$

No real solution

41.
$$\begin{aligned} 2x^2 + 5x - 8 &= 0 \\ 2x^2 + 5x &= 8 \\ x^2 + \frac{5}{2}x &= 4 \\ x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 &= 4 + \left(\frac{5}{4}\right)^2 \\ \left(x + \frac{5}{4}\right)^2 &= \frac{89}{16} \\ x + \frac{5}{4} &= \pm\frac{\sqrt{89}}{4} \\ x &= -\frac{5}{4} \pm \frac{\sqrt{89}}{4} \\ x &= \frac{-5 \pm \sqrt{89}}{4} \end{aligned}$$

42.
$$\begin{aligned} 3x^2 - 4x - 7 &= 0 \\ 3x^2 - 4x &= 7 \\ x^2 - \frac{4}{3}x &= \frac{7}{3} \\ x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 &= \frac{7}{3} + \left(-\frac{2}{3}\right)^2 \\ \left(x - \frac{2}{3}\right)^2 &= \frac{25}{9} \\ x - \frac{2}{3} &= \pm\frac{5}{3} \\ x &= \frac{2}{3} \pm \frac{5}{3} \\ x = -1 \quad &\text{or} \quad x = \frac{7}{3} \end{aligned}$$

43. (a) $y = (x + 3)^2 - 4$

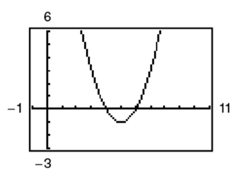


(b) The x-intercepts are $(-1, 0)$ and $(-5, 0)$.

(c)
$$\begin{aligned} 0 &= (x + 3)^2 - 4 \\ 4 &= (x + 3)^2 \\ \pm\sqrt{4} &= x + 3 \\ -3 \pm 2 &= x \\ x &= -1 \text{ or } x = -5 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = (x + 3)^2 - 4$.

44. (a) $y = (x - 5)^2 - 1$

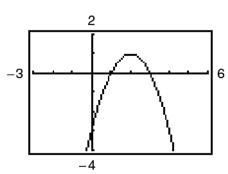


(b) The x-intercepts are $(4, 0)$ and $(6, 0)$.

(c)
$$\begin{aligned} 0 &= (x - 5)^2 - 1 \\ (x - 5)^2 &= 1 \\ x - 5 &= \pm\sqrt{1} \\ x &= 5 \pm 1 = 6, 4 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = (x - 5)^2 - 1$.

45. (a) $y = 1 - (x - 2)^2$

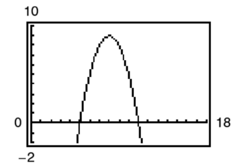


(b) The x-intercepts are $(1, 0)$ and $(3, 0)$.

(c)
$$\begin{aligned} 0 &= 1 - (x - 2)^2 \\ (x - 2)^2 &= 1 \\ x - 2 &= \pm 1 \\ x &= 2 \pm 1 \\ x &= 3 \text{ or } x = 1 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 1 - (x - 2)^2$.

46. (a) $y = 9 - (x - 8)^2$

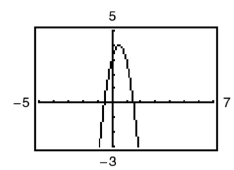


(b) The x-intercepts are $(5, 0)$ and $(11, 0)$.

(c)
$$\begin{aligned} 0 &= 9 - (x - 8)^2 \\ (x - 8)^2 &= 9 \\ x - 8 &= \pm\sqrt{9} \\ x &= 8 \pm 3 = 11, 5 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 9 - (x - 8)^2$.

47. (a) $y = -4x^2 + 4x + 3$

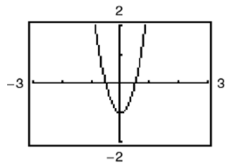


(b) The x-intercepts are $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{3}{2}, 0\right)$.

(c)
$$\begin{aligned} 0 &= -4x^2 + 4x + 3 \\ 4x^2 - 4x &= 3 \\ 4(x^2 - x) &= 3 \\ x^2 - x &= \tfrac{3}{4} \\ x^2 - x + \left(\tfrac{1}{2}\right)^2 &= \tfrac{3}{4} + \left(\tfrac{1}{2}\right)^2 \\ \left(x - \tfrac{1}{2}\right)^2 &= 1 \\ x - \tfrac{1}{2} &= \pm\sqrt{1} \\ x &= \tfrac{1}{2} \pm 1 \\ x &= \tfrac{3}{2} \text{ or } x = -\tfrac{1}{2} \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = -4x^2 + 4x + 3$.

48. (a) $y = 4x^2 - 1$

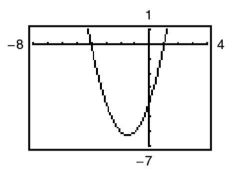


(b) The x-intercepts are $\left(-\frac{1}{2}, 0\right)$ and $\left(\frac{1}{2}, 0\right)$.

(c)
$$0 = 4x^2 - 1$$
$$4x^2 = 1$$
$$x^2 = \frac{1}{4}$$
$$x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 4x^2 - 1$.

49. (a) $y = x^2 + 3x - 4$

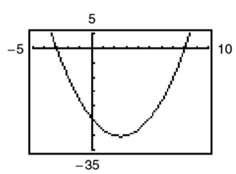


(b) The x-intercepts are $(-4, 0)$ and $(1, 0)$.

(c)
$$0 = x^2 + 3x - 4$$
$$0 = (x + 4)(x - 1)$$
$$x + 4 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = -4 \quad \text{or} \quad x = 1$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = x^2 + 3x - 4$.

50. (a) $y = x^2 - 5x - 24$



(b) The x-intercepts are $(8, 0)$ and $(-3, 0)$.

(c)
$$0 = x^2 - 5x - 24$$
$$(x - 8)(x + 3) = 0$$
$$x - 8 = 0 \Rightarrow x = 8$$
$$x + 3 = 0 \Rightarrow x = -3$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = x^2 - 5x - 24$.

51. $9x^2 + 12x + 4 = 0$

$b^2 - 4ac = (12)^2 - 4(9)(4) = 0$

One repeated real solution

52. $x^2 + 2x + 4 = 0$

$b^2 - 4ac = (2)^2 - 4(1)(4) = 4 - 16 = -12 < 0$

No real solution

53. $2x^2 - 5x + 5 = 0$

$b^2 - 4ac = (-5)^2 - 4(2)(5) = -15 < 0$

No real solution

54. $-5x^2 - 4x + 1 = 0$

$b^2 - 4ac = (-4)^2 - 4(-5)(1) = 16 + 20 = 36 > 0$

Two real solutions

55. $2x^2 - x - 1 = 0$

$b^2 - 4ac = (-1)^2 - 4(2)(-1) = 9 > 0$

Two real solutions

56. $x^2 - 4x + 4 = 0$

$b^2 - 4ac = (-4)^2 - 4(1)(4) = 16 - 16 = 0$

One repeated solution

57. $\frac{1}{3}x^2 - 5x + 25 = 0$

$b^2 - 4ac = (-5)^2 - 4\left(\frac{1}{3}\right)(25) = -\frac{25}{3} < 0$

No real solution

58. $\frac{4}{7}x^2 - 8x + 28$

$b^2 - 4ac = (-8)^2 - 4\left(\frac{4}{7}\right)(28) = 64 - 64 = 0$

One repeated solution

59. $0.2x^2 + 1.2x - 8 = 0$

$b^2 - 4ac = (1.2)^2 - 4(0.2)(-8) = 7.84 > 0$

Two real solutions

60. $9 + 2.4x - 8.3x^2 = 0$

$$b^2 - 4ac = (2.4)^2 - 4(-8.3)(9)$$
$$= 5.76 + 298.8 = 304.56 > 0$$

Two real solutions

61. $2x^2 + x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-1 \pm 3}{4} = \frac{1}{2}, -1$$

62. $2x^2 - x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{1 \pm \sqrt{1 + 8}}{4}$$
$$= \frac{1 \pm 3}{4} = 1, -\frac{1}{2}$$

63. $16x^2 + 8x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-8 \pm \sqrt{8^2 - 4(16)(-3)}}{2(16)}$$
$$= \frac{-8 \pm 16}{32} = \frac{1}{4}, -\frac{3}{4}$$

64. $25x^2 - 20x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-20) \pm \sqrt{(-20)^2 - (25)(3)}}{2(25)}$$
$$= \frac{20 \pm \sqrt{400 - 300}}{50}$$
$$= \frac{20 \pm 10}{50} = \frac{3}{5}, \frac{1}{5}$$

65. $x^2 + 8x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-8 \pm \sqrt{8^2 - 4(1)(-4)}}{2(1)}$$
$$= \frac{-8 \pm 4\sqrt{5}}{2} = -4 \pm 2\sqrt{5}$$

66. $9x^2 + 30x + 25 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-30 \pm \sqrt{30^2 - 4(9)(25)}}{2(9)}$$
$$= \frac{-30 \pm 0}{18} = -\frac{5}{3}$$

67. $2x^2 - 7x + 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(1)}}{2(2)}$$
$$= \frac{7 \pm \sqrt{49 - 8}}{2(2)}$$
$$= \frac{7 \pm \sqrt{41}}{4}$$
$$= \frac{7}{4} \pm \frac{\sqrt{41}}{4}$$

68. $36x^2 + 24x - 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-24 \pm \sqrt{24^2 - 4(36)(-7)}}{2(36)}$$
$$= \frac{-24 \pm \sqrt{576 + 1008}}{72}$$
$$= \frac{-24 \pm \sqrt{(144)(11)}}{72}$$
$$= -\frac{1}{3} \pm \frac{\sqrt{11}}{6}$$

69. $2 + 2x - x^2 = 0$

$-x^2 + 2x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)}$$
$$= \frac{-2 \pm 2\sqrt{3}}{-2}$$
$$= 1 \pm \sqrt{3}$$

70. $x^2 + 10 + 8x = 0$

$$x^2 + 8x + 10 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{-8 \pm \sqrt{64 - 40}}{2(1)}$$

$$= \frac{-8 \pm \sqrt{24}}{2}$$

$$= \frac{-8 \pm 2\sqrt{6}}{2}$$

$$= \frac{2\left(-4 \pm \sqrt{6}\right)}{2}$$

$$= -4 \pm \sqrt{6}$$

71. $x^2 + 16 = -12x$

$$x^2 + 12x + 16 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(12) \pm \sqrt{(12)^2 - 4(1)(16)}}{2(1)}$$

$$= \frac{-12 \pm \sqrt{144 - 64}}{2(1)}$$

$$= \frac{-12 \pm \sqrt{80}}{2}$$

$$= \frac{-12 \pm 4\sqrt{5}}{2}$$

$$= \frac{4\left(-3 \pm \sqrt{5}\right)}{2}$$

$$= 2\left(-3 \pm \sqrt{5}\right) = -6 \pm 2\sqrt{5}$$

72. $4x = 8 - x^2$

$$x^2 + 4x - 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(-8)}}{2(1)}$$

$$= \frac{-4 \pm 4\sqrt{3}}{2} = -2 \pm 2\sqrt{3}$$

73. $4x^2 + 6x = 8$

$$4x^2 + 6x - 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(6) \pm \sqrt{(6)^2 - 4(4)(-8)}}{2(4)}$$

$$= \frac{-6 \pm \sqrt{36 + 128}}{2(4)}$$

$$= \frac{-6 \pm \sqrt{164}}{8}$$

$$= \frac{-6 \pm 2\sqrt{41}}{8}$$

$$= \frac{2\left(-3 \pm \sqrt{41}\right)}{8}$$

$$= \frac{-3 \pm \sqrt{41}}{4}$$

$$= \frac{-3}{4} \pm \frac{\sqrt{41}}{4}$$

74. $16x^2 + 5 = 40x$

$$16x^2 - 40x + 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-40) \pm \sqrt{(-40)^2 - 4(16)(5)}}{2(16)}$$

$$= \frac{40 \pm \sqrt{1600 - 320}}{32}$$

$$= \frac{40 \pm \sqrt{1280}}{32}$$

$$= \frac{40 \pm 16\sqrt{5}}{32}$$

$$= \frac{5}{4} \pm \frac{\sqrt{5}}{2}$$

75. $28x - 49x^2 = 4$

$$-49x^2 + 28x - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-28 \pm \sqrt{28^2 - 4(-49)(-4)}}{2(-49)}$$

$$= \frac{-28 \pm 0}{-98} = \frac{2}{7}$$

76. $3x + x^2 - 1 = 0$

$x^2 + 3x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{13}}{2} = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

77. $8t = 5 + 2t^2$

$-2t^2 + 8t - 5 = 0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{8^2 - 4(-2)(-5)}}{2(-2)}$$

$$= \frac{-8 \pm 2\sqrt{6}}{-4} = 2 \pm \frac{\sqrt{6}}{2}$$

78. $25h^2 + 80h + 61 = 0$

$$h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-80 \pm \sqrt{80^2 - 4(25)(61)}}{2(25)}$$

$$= \frac{-80 \pm \sqrt{6400 - 6100}}{50}$$

$$= -\frac{8}{5} \pm \frac{10\sqrt{3}}{50}$$

$$= -\frac{8}{5} \pm \frac{\sqrt{3}}{5}$$

79. $5.1x^2 - 1.7x - 3.2 = 0$

$$x = \frac{1.7 \pm \sqrt{(-1.7)^2 - 4(5.1)(-3.2)}}{2(5.1)}$$

$\approx 0.976, -0.643$

80. $2x^2 - 2.50x - 0.42 = 0$

$$x = \frac{-(-2.50) \pm \sqrt{(-2.50)^2 - 4(2)(-0.42)}}{2(2)}$$

$$= \frac{2.50 \pm \sqrt{9.61}}{4} = 1.400, -0.150$$

81. $-0.67x^2 + 0.5x + 1.375 = 0$

$$x = \frac{0.5 \pm \sqrt{(0.5)^2 - 4(-0.67)(1.375)}}{2(5.1)}$$

$\approx -1.107, 1.853$

82. $-0.005x^2 + 0.101x - 0.193 = 0$

$$x = \frac{-0.101 \pm \sqrt{(0.101)^2 - 4(-0.005)(-0.193)}}{2(-0.005)}$$

$$= \frac{-0.101 \pm \sqrt{0.006341}}{-0.01}$$

$\approx 2.137, 18.063$

83. $12.67x^2 + 31.55x + 8.09 = 0$

$$x = \frac{-31.55 \pm \sqrt{(31.55)^2 - 4(12.67)(8.09)}}{2(12.67)}$$

$\approx -2.200, -0.290$

84. $-3.22x^2 - 0.08x + 28.651 = 0$

$$x = \frac{-(-0.08) \pm \sqrt{(-0.08)^2 - 4(-3.22)(28.651)}}{2(-3.22)}$$

$$= \frac{0.08 \pm \sqrt{369.031}}{-6.44} \approx -2.995, 2.971$$

85. $x^2 - 2x - 1 = 0$ Complete the square.

$x^2 - 2x = 1$

$x^2 - 2x + 1^2 = 1 + 1^2$

$(x - 1)^2 = 2$

$x - 1 = \pm\sqrt{2}$

$x = 1 \pm \sqrt{2}$

86. $14x^2 + 42x = 0$ Factor.

$14x(x + 3x) = 0$

$x(x + 3) = 0$

$x = 0$ or $x + 3 = 0$

$x = -3$

87. $(x + 2)^2 = 64$ Extract square roots.

$x + 2 = \pm 8$

$x + 2 = 8$ or $x + 2 = -8$

$x = 6$ or $x = -10$

88. $x^2 - 14x + 49 = 0$ Extract square roots.

$(x - 7)^2 = 0$

$x - 7 = 0$

$x = 7$

89. $x^2 - x - \frac{11}{4} = 0$ Complete the square.

$$x^2 - x = \frac{11}{4}$$
$$x^2 - x + \left(\frac{1}{2}\right)^2 = \frac{11}{4} + \left(\frac{1}{2}\right)^2$$
$$\left(x - \frac{1}{2}\right)^2 = \frac{12}{4}$$
$$x - \frac{1}{2} = \pm\sqrt{\frac{12}{4}}$$
$$x = \frac{1}{2} \pm \sqrt{3}$$

90. $x^2 + 3x - \frac{3}{4} = 0$ Complete the square.

$$x^2 + 3x + \left(\frac{3}{2}\right)^2 = \frac{3}{4} + \frac{9}{4}$$
$$\left(x + \frac{3}{2}\right)^2 = 3$$
$$x + \frac{3}{2} = \pm\sqrt{3}$$
$$x = -\frac{3}{2} \pm \sqrt{3}$$

91. $3x + 4 = 2x^2 - 7$ Quadratic Formula

$$0 = 2x^2 - 3x - 11$$
$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-11)}}{2(2)}$$
$$= \frac{3 \pm \sqrt{97}}{4}$$
$$= \frac{3}{4} \pm \frac{\sqrt{97}}{4}$$

92. $(x + 1)^2 = x^2$ Extract square roots.

$$x^2 = (x + 1)^2$$
$$x = \pm(x + 1)$$

For $x = +(x + 1)$:

$0 \neq 1$ No solution

For $x = -(x + 1)$:

$$2x = -1$$
$$x = -\frac{1}{2}$$

93. $$\frac{1}{x^2 - 2x + 5} = \frac{1}{x^2 - 2x + 1^2 + 5} = \frac{1}{(x - 1)^2 + 4}$$

94. $$\frac{1}{x^2 + 6x + 10} = \frac{1}{x^2 + 6x + (3)^2 - (3)^2 + 10} = \frac{1}{x^2 + 6x + 9 + 1} = \frac{1}{(x + 3)^2 + 1}$$

95. $$\frac{4}{x^2 + 10x + 74} = \frac{4}{x^2 + 10x + (5)^2 - (5)^2 + 74} = \frac{4}{x^2 + 10x + 25 + 49} = \frac{4}{(x + 5)^2 + 49}$$

96. $$\frac{5}{x^2 - 18x + 162} = \frac{5}{x^2 - 18x + (9)^2 - (9)^2 + 162} = \frac{5}{x^2 - 18x + 81 + 81} = \frac{5}{(x - 9)^2 + 81}$$

97. $$\frac{1}{\sqrt{3 + 2x - x^2}} = \frac{1}{\sqrt{-1(x^2 - 2x - 3)}} = \frac{1}{\sqrt{-1\left[x^2 - 2x + (1)^2 - (1)^2 - 3\right]}} = \frac{1}{\sqrt{-1(x^2 - 2x + 1) + 4}} = \frac{1}{\sqrt{4 - (x - 1)^2}}$$

98. $$\frac{1}{\sqrt{9 + 8x - x^2}} = \frac{1}{\sqrt{-1(x^2 - 8x - 9)}} = \frac{1}{\sqrt{-1\left[x^2 - 8x + (4)^2 - (4)^2 - 9\right]}} = \frac{1}{\sqrt{-1\left[(x^2 - 8x + 16) - 25\right]}} = \frac{1}{\sqrt{25 - (x - 4)^2}}$$

99. $$\frac{1}{\sqrt{12 + 4x - x^2}} = \frac{1}{\sqrt{-1(x^2 - 4x - 12)}} = \frac{1}{\sqrt{-1\left[x^2 - 4x + (2)^2 - (2)^2 - 12\right]}}$$
$$= \frac{1}{\sqrt{-1\left[(x^2 - 4x + 4) - 16\right]}} = \frac{1}{\sqrt{16 - (x - 2)^2}}$$

100. $\dfrac{1}{\sqrt{16 - 6x - x^2}} = \dfrac{1}{\sqrt{16 - 1(x^2 + 6x)}}$

$= \dfrac{1}{\sqrt{16 - (x^2 + 6x + 3^2) + 9}}$

$= \dfrac{1}{\sqrt{25 - (x + 3)^2}}$

101. (a) $w(w + 14) = 1632$

(b) $w^2 + 14w - 1632 = 0$

$(w + 48)(w - 34) = 0$

$w = -48$ or $w = 34$

Because the width must be greater than zero, $w = 34$ feet and the length is $w + 14 = 48$ feet.

102. Total fencing: $4x + 3y = 100$

Total area: $2xy = 350$

$x = \dfrac{100 - 3y}{4}$

$2\left(\dfrac{100 - 3y}{4}\right)y = 350$

$\dfrac{1}{2}(100y - 3y^2) - 350 = 0$

$100y - 3y^2 - 700 = 0$

$-3y^2 + 100y - 700 = 0$

$(3y - 70)(-y + 10) = 0$

$3y - 70 = 0 \Rightarrow y = \dfrac{70}{3}$

$-y + 10 = 0 \Rightarrow y = 10$

For $y = \dfrac{70}{3}$:

$2x\left(\dfrac{70}{3}\right) = 350$

$x = 7.5$

For $y = 10$:

$2x(10) = 350$

$x = 17.5$

There are two solutions: $x = 7.5$ meters and $y = 23\frac{1}{3}$ meters or $x = 17.5$ meters and $y = 10$ meters.

103. $S = x^2 + 4xh$

$108 = x^2 + 4x(3)$

$0 = x^2 + 12x - 108$

$0 = (x + 18)(x - 6)$

$x = -18$ or $x = 6$

Because x must be positive, $x = 6$ inches. The dimensions of the box are 6 inches $\times$ 6 inches $\times$ 3 inches.

104. Volume: $4x^2 = 576$

$x^2 = 144$

$x = \pm 12$

Because x must be positive, $x = 12$ centimeters and the side length of the original material is $x + 8 = 20$ centimeters. The dimensions of the original material are 20 centimeters $\times$ 20 centimeters.

105. (a) Volume is 1024 cubic feet.

$V = l \cdot w \cdot h$

$= x(x + 1)(4)$

$= 4x^2 + 4x$

So, $4x^2 + 4x = 1024$

$4x^2 + 4x - 1024 = 0$

$4(x^2 + x - 256) = 0$

$x^2 + x - 256 = 0$

$x = \dfrac{-1 \pm \sqrt{1^2 - 4(1) - 256}}{2(1)}$

$x \approx 15.51$

$x + 1 \approx 16.51$

So the base of the pool is approximately 15.51 feet $\times$ 16.51 feet.

(b) Because 1 cubic foot of water weighs approximately 62.4 pounds,

$1024 \text{ cubic feet} \cdot \dfrac{62.4 \text{ pounds}}{1 \text{ cubic foot}} = 63{,}897.6 \text{ pounds.}$

106. Original arrangement: x rows, y seats per row,

$xy = 72, y = \dfrac{72}{x}$

New arrangement: $(x - 2)$ rows, $(y + 3)$ seats per row

$(x - 2)(y + 3) = 72$

$(x - 2)\left(\dfrac{72}{x} + 3\right) = 72$

$x(x - 2)\left(\dfrac{72}{x} + 3\right) = 72x$

$(x - 2)(72 + 3x) = 72x$

$72x + 3x^2 - 144 - 6x = 72x$

$3x^2 - 6x - 144 = 0$

$x^2 - 2x - 48 = 0$

$(x - 8)(x + 6) = 0$

$x - 8 = 0 \Rightarrow x = 8$

$x + 6 = 0 \Rightarrow x = -6$

Originally, there were 8 rows of seats with $\dfrac{72}{9} = 9$ seats per row.

107. (a) $s = -16t^2 + v_0t + s_0$

Since the object was dropped, $v_0 = 0$, and the initial height is $s_0 = 984$. Thus,

$s = -16t^2 + 984.$

(b) $s = -16(4)^2 + 984 = 728$ feet

(c) $0 = -16t^2 + 984$

$16t^2 = 984$

$t^2 = \dfrac{984}{16}$

$t = \sqrt{\dfrac{984}{16}} = \dfrac{\sqrt{246}}{2} \approx 7.84$

It will take the coin about 7.84 seconds to strike the ground.

108. (a) $s = -16t^2 + v_0t + s_0$

$s = -16t^2 + 550$

Let $s = 0$ and solve for t.

$0 = -16t^2 + 550$

$16t^2 = 550$

$t^2 = \dfrac{550}{16}$

$t = \sqrt{\dfrac{550}{16}}$

$t \approx 5.86$

The supply package will take about 5.86 seconds to reach the ground.

(b) *Verbal Model:* (Distance) = (Rate) · (Time)

Labels: Distance $= d$

Rate = 138 miles per hour

Time = $\dfrac{5.86 \text{ seconds}}{3600 \text{ seconds per hour}}$

≈ 0.0016 hour

Equation: $d = (138)(0.0016) \approx 0.22$ mile

The supply package will travel about 0.2 mile.

109. (a) $s = -16t^2 + v_0t + s_0$

Since the object was dropped, $v_0 = 0$, and the initial height is $s_0 = 1815$. Thus, $s = -16t^2 + 1815$.

(b)

Time, t	0	2	4	6	8	10	12
Height, s	1815	1751	1559	1239	791	215	−489

(c) The object reaches the ground between $t = 10$ seconds and $t = 12$ seconds. Numerical approximation will vary, though 10.7 seconds is a reasonable estimate.

(d) $0 = -16t^2 + 1815$

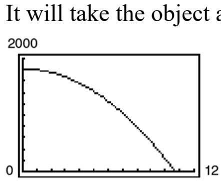
$16t^2 = 1815$

$t^2 = \dfrac{1815}{16}$

$t = \sqrt{\dfrac{1815}{16}} = \dfrac{11\sqrt{15}}{4} \approx 10.65$

It will take the object about 10.65 seconds to reach the ground.

(e)



The zero of the graph is at $t \approx 10.65$ seconds.

110. (a) $s = -16t^2 + v_0 t + s_0$

$v_0 = 100 \text{ mph} = \dfrac{(100)(5280)}{3600} = 146\dfrac{2}{3} \text{ ft/sec}$

$s_0 = 6 \text{ feet } 3 \text{ inches} = 6\dfrac{1}{4} \text{ feet}$

$s = -16t^2 + 146\dfrac{2}{3}t + 6\dfrac{1}{4}$

$s = -16t^2 + 146\dfrac{2}{3}t + 6.25$

(b) When $t = 4$: $s(4) \approx 336.92$ feet

When $t = 5$: $s(5) \approx 339.58$ feet

When $t = 6$: $s(6) \approx 310.25$ feet

During the interval $4 \le t \le 6$, the baseball reached its maximum height.

(c) The ball hits the ground when $s = 0$.

$-16t^2 + 146\frac{2}{3}t + 6\frac{1}{4} = 0$

Using the Quadratic Formula, $t = \dfrac{-146\frac{2}{3} \pm \sqrt{\left(146\frac{2}{3}\right)^2 - 4(-16)\left(6\frac{1}{4}\right)}}{2(-16)} \approx \dfrac{-146\frac{2}{3} \pm 148.02}{-32}$,

$t \approx -0.042$ or $t \approx 9.209$. Time is always positive, so the ball will be in the air for approximately 9.209 seconds.

111. $D = 0.0534t^2 - 0.855t + 18.87, 14 \le t \le 20$

(a)

t	14	15	16	17	18	19	20
D	17.37	18.06	18.86	19.77	20.78	21.90	23.13

The public debt reached \$20 trillion sometime in 2017.

(b) $D = 0.0534t^2 - 0.855t + 18.87$

$20 = 0.0534t^2 - 0.855t - 1.13$

$0 = 0.0534t^2 - 0.855t - 1.13$

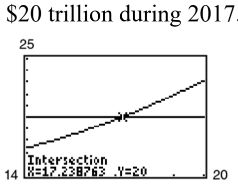
$t = \dfrac{\left[0.855 \pm \sqrt{(-0.855)^2 - 4(0.0534)(-1.13)}\right]}{2(0.0534)}$

$= \dfrac{\left(0.855 \pm \sqrt{0.972393}\right)}{0.1068}$

$t \approx -1.23$ and $t \approx 17.24$

Because the domain of the model is $14 \le t \le 20$, $t \approx 17.24$ is the only solution. So, the total public debt reached \$20 trillion during 2017.

(c)

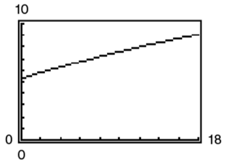


The graphs of D and $y = 20$ intersect at $t \approx 17.24$.

So, the total public debt reached \$20 trillion during 2017.

112. $P = -0.0025t^2 + 0.254t + 5.30, 1 \le t \le 18$

(a)



Using the graph, the average ticket price reached $7.00 when $t \approx 7.2$, which correspond to 2007.

(b) $P = -0.0025t^2 + 0.254t + 5.30$

$7 = -0.0025t^2 + 0.254t + 5.30$

$0 = -0.0025t^2 + 0.254t - 1.70$

$$t = \frac{\left[-0.254 \pm \sqrt{(0.254)^2 - 4(-0.0025)(-1.70)}\right]}{2(-0.0025)}$$

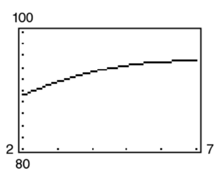
$$= \frac{\left[-0.254 \pm \sqrt{0.047516}\right]}{(-0.005)}$$

$$= \frac{[-0.254 \pm 0.21798]}{(-0.005)}$$

$t \approx 7.204$ and 94.396

Because the domain of the model is $1 \le t \le 18$, $t \approx 7.204$ is the only solution. So, the average ticket price reached $7.00 during 2007.

113. (a) $L = -0.270t^2 + 3.59t + 83.1, \ 2 \le t \le 7$



Minimum at $t = 2$ (89.2% at 2 P.M.)

Maximum at $t \approx 6.65$ (95.0% at 6:38 P.M.)

(b) $L = -0.270t^2 + 3.59t + 83.1$

$93 = -0.270t^2 + 3.59t + 83.1$

$0 = -0.270t^2 + 3.59t - 9.9$

$0 = 0.270t^2 - 3.59t + 9.9$

Using the Quadratic Formula,

$$t = \frac{-(-3.59) \pm \sqrt{(-3.59)^2 - 4(0.270)(9.9)}}{2(0.270)} = \frac{3.59 \pm \sqrt{2.1961}}{0.54},$$

$t \approx 3.9$ and $t \approx 9.4$

Because the domain of the model is $2 \le t \le 7$, $t \approx 3.9$ is the only solution. The patient's blood oxygen level was 93% at approximately 4:00 P.M.

114. (a) $150 = 0.45x^2 - 1.65x + 50.75$

$0 = 0.45x^2 - 1.65x - 99.25$

$$x = \frac{1.65 \pm \sqrt{(-1.65)^2 - 4(0.45)(-99.25)}}{2(0.45)}$$

$\approx -13.1, 16.8$

Because $10 \le x \le 25$, choose 16.8°C.

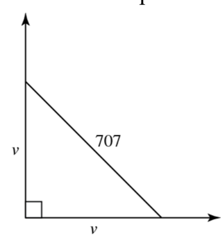
$x = 10: 0.45(10)^2 - 1.65(10) + 50.75 = 79.25$

(b) $x = 20: 0.45(20)^2 - 1.65(20) + 50.75 = 197.75$

$197.75 \div 79.25 \approx 2.5$

Oxygen consumption is increased by a factor of approximately 2.5.

115. Let v be the speed of the two planes. After 1 hour, the planes have traveled v miles.



So, $2v^2 = 707^2 \Rightarrow v = \dfrac{707}{\sqrt{2}} \approx 500.$

The speed is approximately 500 miles per hour.

116. (a) *Model:* $(\text{winch})^2 + (\text{distance to dock})^2 = (\text{length of rope})^2$

Labels: winch $= 15$, distance to dock $= x$, length of rope $= l$

Equation: $15^2 + x^2 = l^2$

(b) When $l = 75$: $15^2 + x^2 = 75^2$

$x^2 = 5625 - 225 = 5400$

$x = \sqrt{5400} = 30\sqrt{6} \approx 73.5$

The boat is approximately 73.5 feet from the dock when there is 75 feet of rope out.

117. $-3x^2 + x = -5$

$-3x^2 + x + 5 = 0$

$b^2 - 4ac = (1)^2 - 4(-3)(5) = 1 + 60 = 61 > 0.$

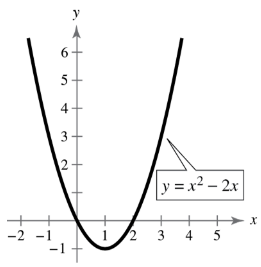
True. The quadratic equation has two real solutions.

118. False. The product must equal zero for the Zero Factor Property to be used.

119. Yes, the vertex of the parabola would be on the x-axis.

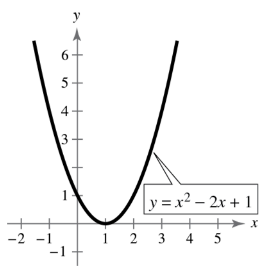
120. (a) The discriminant is positive because the graph has two x-intercepts. $y = x^2 - 2x$

$$b^2 = 4ac = (-2)^2 - 4(1)(0) = 4$$



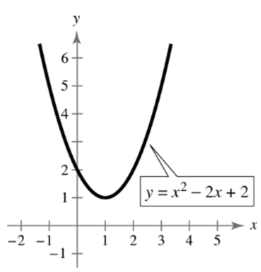
(b) The discriminant is zero because the graph has one x-intercept. $y = x^2 - 2x + 1$

$$b^2 = 4ac = (-2)^2 - 4(1)(1) = 0$$



(c) The discriminant is negative because the graph has no x-intercepts. $y = x^2 - 2x + 2$

$$b = 4ac = (-2)^2 - 4(1)(2) = -4$$



121. The equation in general form is $3x^2 + x - 3 = 0$, so $a = 3, b = 1, c = -3$.

122. The factors of the quadratic equation should be $(x - 2)$ and $(x - 4)$.

$$(x - 2)(x - 4) = 0 \Rightarrow x^2 - 6x + 8 = 0$$

123. *Sample answer:* $(x - 0)(x - 4) = 0$

$$x(x - 5) = 0$$
$$x^2 - 4x = 0$$

124. *Sample answer:* $(x - (-2))(x - (-8)) = 0$

$$(x + 2)(x + 8) = 0$$
$$x^2 + 10x + 16 = 0$$

125. One possible equation is:

$$(x - 8)(x - 14) = 0$$
$$x^2 - 22x + 112 = 0$$

Any non-zero multiple of this equation would also have these solutions.

126. $x = \frac{1}{6} \Rightarrow 6x = 1 \Rightarrow 6x - 1$ is a factor.

$x = -\frac{2}{5} \Rightarrow 5x = -2 \Rightarrow 5x + 2$ is a factor.

$$(6x - 1)(5x + 2) = 0$$
$$30x^2 + 7x - 2 = 0$$

127. One possible equation is:

$$\left[x - \left(1 + \sqrt{2}\right)\right]\left[x - \left(1 - \sqrt{2}\right)\right] = 0$$
$$\left[(x - 1) - \sqrt{2}\right]\left[(x - 1) + \sqrt{2}\right] = 0$$
$$(x - 1)^2 - \left(\sqrt{2}\right)^2 = 0$$
$$x^2 - 2x + 1 - 2 = 0$$
$$x^2 - 2x - 1 = 0$$

Any non-zero multiple of this equation would also have these solutions.

128. $x = -3 + \sqrt{5}, x = -3 - \sqrt{5}$, so:

$$\left(x - \left(-3 + \sqrt{5}\right)\right)\left(x - \left(-3 - \sqrt{5}\right)\right) = 0$$
$$\left(x + 3 - \sqrt{5}\right)\left(x + 3 + \sqrt{5}\right) = 0$$
$$x^2 + 6x + 4 = 0$$

129. $(x^2 + 5x + 11) + (2x^2 - 13x + 16) = 3x^2 - 8x + 27$

130. $(7x^2 - 8x + 4) + (9x^3 + 3x^2 + x) = 9x^3 + 10x^2 - 7x + 4$

131. $(12x^2 - 15) - (x^2 - 19x - 5) = 12x^2 - 15 - x^2 + 19x + 5 = 11x^2 + 19x - 10$

132. $(x^2 - 3x - 2) - (x^2 - 2) - (x - 3) = x^2 - 3x - 2 - x^2 + 2 - x + 3 = -4x + 3$

133. $(x + 6)(3x - 5) = 3x^2 + 18x - 5x - 30 = 3x^2 + 13x - 30$

134. $(3x + 13)(4x - 7) = 12x^2 + 52x - 21x - 91 = 12x^2 + 31x - 91$

135. $(2x - 9)(2x + 9) = (2x)^2 - 9^2 = 4x^2 - 81$

136. $(4x + 1)^2 = (4x + 1)(4x + 1) = (4x)^2 + 2(4x)(1) + 1^2 = 16x^2 + 8x + 1$

137. $(2x^2 - y)(3x^2 + 4y) = 6x^4 - 3x^2y + 8x^2y - 4y^2 = 6x^4 + 5x^2y - 4y^2$

138. $(4x^3 + 7y^2)(3x^3 - y^2) = 12x^6 + 21x^3y^2 - 4x^3y^2 - 7y^4 = 12x^6 + 17x^3y^2 - 7y^4$

139. $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2$
$= 3 - 2 = 1$

140. $(\sqrt{5} - 1)(\sqrt{5} + 1) = (\sqrt{5})^2 - 1^2 = 5 - 1 = 4$

141. $(7\sqrt{2} - 4 + \sqrt{5})(7\sqrt{2} + 4 - \sqrt{5})$
$= [7\sqrt{2} - (4 - \sqrt{5})][7\sqrt{2} + (4 - \sqrt{5})]$
$= (7\sqrt{2})^2 - (4 - \sqrt{5})^2$
$= 98 - (16 - 8\sqrt{5} + 5)$
$= 77 + 8\sqrt{5}$

142. $(2\sqrt{3} + 3\sqrt{2})(2\sqrt{3} - 3\sqrt{2}) = (2\sqrt{3})^2 - (3\sqrt{2})^2$
$= 12 - 18 = -6$

143. $\dfrac{12}{5\sqrt{3}} = \dfrac{12}{5\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{12\sqrt{3}}{5 \cdot 3} = \dfrac{4\sqrt{3}}{5}$

144. $\dfrac{4}{\sqrt{10} - 2} \cdot \dfrac{\sqrt{10} + 2}{\sqrt{10} + 2} = \dfrac{4(\sqrt{10} + 2)}{10 - 4} = \dfrac{4\sqrt{10} + 8}{6}$
$= \dfrac{2\sqrt{10} + 4}{3}$

145. $\dfrac{3}{8 + \sqrt{11}} = \dfrac{3}{8 + \sqrt{11}} \cdot \dfrac{8 - \sqrt{11}}{8 - \sqrt{11}}$
$= \dfrac{3(8 - \sqrt{11})}{64 - 11}$
$= \dfrac{24 - 3\sqrt{11}}{53}$

146. $\dfrac{14}{3\sqrt{10} - 1} = \dfrac{14}{3\sqrt{10} - 1} \cdot \dfrac{3\sqrt{10} + 1}{3\sqrt{10} + 1}$
$= \dfrac{14(3\sqrt{10} + 1)}{90 - 1}$
$= \dfrac{14 + 42\sqrt{10}}{89}$

Section 1.5 Complex Numbers

1. $\sqrt{-1}$; -1

2. principal square

3. (a) ii (b) iii (c) i

4. To multiply two complex numbers, $(a + bi)(c + di)$, the FOIL Method can be used;
$(a + bi)(c + di) = ac + adi + bci + bdi^2$
$= (ac - bd) + (ad + bc)i.$

5. The additive inverse of $2 - 4i$ is $-2 + 4i$.

6. The Complex conjugate of $2 - 4i$ is $2 + 4i$.

7. $a + bi = 9 + 8i$
$a = 9$
$b = 8$

8. $a + bi = b + (2a - 1)i$
$a = b$
$b = 2a - 1 = 2b - 1 \Rightarrow b = 1$ and $a = 1$

9. $(5 + i) + (2 + 3i) = 5 + i + 2 + 3i$
$= 7 + 4i$

10. $(13 - 2i) + (-5 + 6i) = 8 + 4i$

11. $(9 - i) - (8 - i) = 1$

12. $(3 + 2i) - (6 + 13i) = 3 + 2i - 6 - 13i$
$= -3 - 11i$

13. $(-2 + \sqrt{-8}) + (5 - \sqrt{-50}) = -2 + 2\sqrt{2}i$
$+ 5 - 5\sqrt{2}i = 3 - 3\sqrt{2}i$

14. $\left(8 + \sqrt{-18}\right) - \left(4 + 3\sqrt{2}i\right) = 8 + 3\sqrt{2}\,i$
$- 4 - 3\sqrt{2}\,i = 4$

15. $13i - (14 - 7i) + (2 - 11i) = -14 + 2 + (13 + 7 - 11)i = -12 + 9i$

16. $(25 + 6i) + (-10 + 11i) - (17 - 15i) = (25 - 10 - 17) + (6 + 11 + 15)i = -2 + 32i$

17. $(1 + i)(3 - 2i) = 3 - 2i + 3i - 2i^2$
$= 3 + i + 2 = 5 + i$

18. $(7 - 2i)(3 - 5i) = 21 - 35i - 6i + 10i^2$
$= 21 - 41i - 10$
$= 11 - 41i$

19. $12i(1 - 9i) = 12i - 108i^2$
$= 12i + 108$
$= 108 + 12i$

20. $-8i(9 + 4i) = -72i - 32i^2$
$= 32 - 72i$

21. $\left(\sqrt{2} + 3i\right)\left(\sqrt{2} - 3i\right) = 2 - 9t^2$
$= 2 + 9 = 11$

22. $\left(4 + \sqrt{7}i\right)\left(4 - \sqrt{7}i\right) = 16 - 7i^2$
$= 16 + 7 = 23$

23. $(6 + 7i)^2 = 36 + 84i + 49i^2$
$= 36 + 84i - 49$
$= -13 + 84i$

24. $(5 - 4i)^2 = 25 - 40i + 16i^2$
$= 25 - 40i - 16$
$= 9 - 40i$

25. The complex conjugate of $9 + 2i$ is $9 - 2i$.
$(9 + 2i)(9 - 2i) = 81 - 4i^2$
$= 81 + 4$
$= 85$

26. The complex conjugate of $8 - 10i$ is $8 + 10i$.
$(8 - 10i)(8 + 10i) = 64 - 100i^2$
$= 64 + 100$
$= 164$

27. The complex conjugate of $-1 - \sqrt{5}i$ is $-1 + \sqrt{5}i$.
$\left(-1 - \sqrt{5}i\right)\left(-1 + \sqrt{5}i\right) = 1 - 5i^2$
$= 1 + 5 = 6$

28. The complex conjugate of $-3 + \sqrt{2}i$ is $-3 - \sqrt{2}i$.
$\left(-3 + \sqrt{2}i\right)\left(-3 - \sqrt{2}i\right) = 9 - 2i^2$
$= 9 + 2$
$= 11$

29. The complex conjugate of $\sqrt{-20} = 2\sqrt{5}i$ is $-2\sqrt{5}i$.
$\left(2\sqrt{5}i\right)\left(-2\sqrt{5}i\right) = -20i^2 = 20$

30. The complex conjugate of $\sqrt{-15} = \sqrt{15}i$ is $-\sqrt{15}i$.
$\left(\sqrt{15}i\right)\left(-\sqrt{15}i\right) = -15i^2 = 15$

31. The complex conjugate of $1 - \sqrt{-6}$ is $1 + \sqrt{-6}$.
$\left(1 - \sqrt{-6}\right)\left(1 + \sqrt{-6}\right) = 1 - (-6) = 7$

32. The complex conjugate of $1 + \sqrt{-8}$ is $1 - \sqrt{-8}$.
$\left(1 + \sqrt{-8}\right)\left(1 - \sqrt{-8}\right) = 1 - (-8) = 9$

33. $\dfrac{2}{4 - 5i} = \dfrac{2}{4 - 5i} \cdot \dfrac{4 + 5i}{4 + 5i}$
$= \dfrac{2(4 + 5i)}{16 + 25} = \dfrac{8 + 10i}{41} = \dfrac{8}{41} + \dfrac{10}{41}i$

34. $\dfrac{13}{1 - i} \cdot \dfrac{(1 + i)}{(1 + i)} = \dfrac{13 + 13i}{1 - i^2} = \dfrac{13 + 13i}{2} = \dfrac{13}{2} + \dfrac{13}{2}i$

35. $\dfrac{5 + i}{5 - i} \cdot \dfrac{(5 + i)}{(5 + i)} = \dfrac{25 + 10i + i^2}{25 - i^2}$
$= \dfrac{24 + 10i}{26} = \dfrac{12}{13} + \dfrac{5}{13}i$

36. $\dfrac{6 - 7i}{1 - 2i} \cdot \dfrac{1 + 2i}{1 + 2i} = \dfrac{6 + 12i - 7i - 14i^2}{1 - 4i^2}$
$= \dfrac{20 + 5i}{5} = 4 + i$

37. $\dfrac{9 - 4i}{i} \cdot \dfrac{-i}{-i} = \dfrac{-9i + 4i^2}{-i^2} = -4 - 9i$

38. $\dfrac{8 + 16i}{2i} \cdot \dfrac{-2i}{-2i} = \dfrac{-16i - 32i^2}{-4i^2} = 8 - 4i$

39. $\dfrac{3i}{(4-5i)^2} = \dfrac{3i}{16-40i+25i^2} = \dfrac{3i}{-9-40i}\cdot\dfrac{-9+40i}{-9+40i}$

$= \dfrac{-27i+120i^2}{81+1600} = \dfrac{-120-27i}{1681}$

$= -\dfrac{120}{1681} - \dfrac{27}{1681}i$

40. $\dfrac{5i}{(2+3i)^2} = \dfrac{5i}{4+12i+9i^2}$

$= \dfrac{5i}{-5+12i}\cdot\dfrac{-5-12i}{-5-12i}$

$= \dfrac{-25i-60i^2}{25-144i^2}$

$= \dfrac{60-25i}{169} = \dfrac{60}{169} - \dfrac{25}{169}i$

41. $\sqrt{-6}\cdot\sqrt{-2} = \left(\sqrt{6}i\right)\left(\sqrt{2}i\right) = \sqrt{12}i^2 = \left(2\sqrt{3}\right)(-1)$

$= -2\sqrt{3}$

42. $\sqrt{-5}\cdot\sqrt{-10} = \left(\sqrt{5}i\right)\left(\sqrt{10}i\right)$

$= \sqrt{50}i^2 = 5\sqrt{2}(-1) = -5\sqrt{2}$

43. $\left(\sqrt{-15}\right)^2 = \left(\sqrt{15}i\right)^2 = 15i^2 = -15$

44. $\left(\sqrt{-75}\right)^2 = \left(\sqrt{75}i\right)^2 = 75i^2 = -75$

45. $\sqrt{-8} + \sqrt{-50} = \sqrt{8}\,i + \sqrt{50}\,i$

$= 2\sqrt{2}\,i + 5\sqrt{2}\,i$

$= 7\sqrt{2}\,i$

46. $\sqrt{-45} - \sqrt{-5} = \sqrt{45}\,i - \sqrt{5}\,i$

$= 3\sqrt{5}\,i - \sqrt{5}\,i$

$= 2\sqrt{5}\,i$

47. $\left(3+\sqrt{-5}\right)\left(7-\sqrt{-10}\right) = \left(3+\sqrt{5}i\right)\left(7-\sqrt{10}i\right)$

$= 21 - 3\sqrt{10}i + 7\sqrt{5}i - \sqrt{50}i^2$

$= \left(21+\sqrt{50}\right) + \left(7\sqrt{5} - 3\sqrt{10}\right)i$

$= \left(21+5\sqrt{2}\right) + \left(7\sqrt{5} - 3\sqrt{10}\right)i$

48. $\left(2-\sqrt{-6}\right)^2 = \left(2-\sqrt{6}i\right)\left(2-\sqrt{6}i\right)$

$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6i^2$

$= 4 - 2\sqrt{6}i - 2\sqrt{6}i + 6(-1)$

$= 4 - 6 - 4\sqrt{6}i$

$= -2 - 4\sqrt{6}i$

49. $x^2 - 2x + 2 = 0;\ a = 1,\ b = -2,\ c = 2$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$

$= \dfrac{2 \pm \sqrt{-4}}{2}$

$= \dfrac{2 \pm 2i}{2}$

$= 1 \pm i$

50. $x^2 + 6x + 10 = 0;\ a = 1,\ b = 6,\ c = 10$

$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)}$

$= \dfrac{-6 \pm \sqrt{-4}}{2}$

$= \dfrac{-6 + 2i}{2}$

$= -3 \pm i$

51. $4x^2 + 16x + 17 = 0;\ a = 4,\ b = 16,\ c = 17$

$x = \dfrac{-16 \pm \sqrt{(16)^2 - 4(4)(17)}}{2(4)}$

$= \dfrac{-16 \pm \sqrt{-16}}{8}$

$= \dfrac{-16 \pm 4i}{8}$

$= -2 \pm \dfrac{1}{2}i$

52. $9x^2 - 6x + 37 = 0;\ a = 9,\ b = -6,\ c = 37$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(37)}}{2(9)}$$
$$= \frac{6 \pm \sqrt{-1296}}{18}$$
$$= \frac{6 \pm 36i}{18} = \frac{1}{3} \pm 2i$$

53. $4x^2 + 16x + 21 = 0;\ a = 4,\ b = 16,\ c = 21$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(21)}}{2(4)}$$
$$= \frac{-16 \pm \sqrt{-80}}{8}$$
$$= \frac{-16 \pm \sqrt{80}\,i}{8}$$
$$= \frac{-16 \pm 4\sqrt{5}\,i}{8}$$
$$= -2 \pm \frac{\sqrt{5}}{2}i$$

54. $16t^2 - 4t + 3 = 0;\ a = 16,\ b = -4,\ c = 3$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)}$$
$$= \frac{4 \pm \sqrt{-176}}{32}$$
$$= \frac{4 \pm 4\sqrt{11}i}{32}$$
$$= \frac{1}{8} \pm \frac{\sqrt{11}}{8}i$$

55. $\frac{3}{2}x^2 - 6x + 9 = 0$ Multiply both sides by 2.

$3x^2 - 12x + 18 = 0;\ a = 3,\ b = -12,\ c = 18$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(18)}}{2(3)}$$
$$= \frac{12 \pm \sqrt{-72}}{6}$$
$$= \frac{12 \pm 6\sqrt{2}i}{6}$$
$$= 2 \pm \sqrt{2}i$$

56. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$ Multiply both sides by 16.

$14x^2 - 12x + 5 = 0;\ a = 14,\ b = -12,\ c = 5$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(14)(5)}}{2(14)}$$
$$= \frac{12 \pm \sqrt{-136}}{28}$$
$$= \frac{12 \pm 2\sqrt{34}i}{28}$$
$$= \frac{3}{7} \pm \frac{\sqrt{34}}{14}i$$

57. $1.4x^2 - 2x + 10 = 0 \Rightarrow 14x^2 - 20x + 100 = 0;$
$a = 14,\ b = -20,\ c = 100$

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(14)(100)}}{2(14)}$$
$$= \frac{20 \pm \sqrt{-5200}}{28}$$
$$= \frac{20 \pm 20\sqrt{13}\,i}{28}$$
$$= \frac{20}{28} \pm \frac{20\sqrt{13}\,i}{28}$$
$$= \frac{5}{7} \pm \frac{5\sqrt{13}}{7}i$$

58. $4.5x^2 - 3x + 12 = 0;\ a = 4.5,\ b = -3,\ c = 12$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4.5)(12)}}{2(4.5)}$$
$$= \frac{3 \pm \sqrt{-207}}{9}$$
$$= \frac{3 \pm 3\sqrt{23}i}{9}$$
$$= \frac{1}{3} \pm \frac{\sqrt{23}}{3}i$$

59. $z_1 = 5 + 2i$

$z_2 = 3 - 4i$

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{5 + 2i} + \frac{1}{3 - 4i}$$

$$= \frac{(3 - 4i) + (5 + 2i)}{(5 + 2i)(3 - 4i)}$$

$$= \frac{8 - 2i}{23 - 14i}$$

$$z = \frac{23 - 14i}{8 - 2i}\left(\frac{8 + 2i}{8 + 2i}\right)$$

$$= \frac{212 - 66i}{68} \approx 3.118 - 0.971i$$

60. $z_1 = 9 + 16i, z_2 = 20 - 10i$

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{9 + 16i} + \frac{1}{20 - 10i}$$

$$= \frac{20 - 10i + 9 + 16i}{(9 + 16i)(20 - 10i)} = \frac{29 + 6i}{340 + 230i}$$

$$z = \left(\frac{340 + 230i}{29 + 6i}\right)\left(\frac{29 - 6i}{29 - 6i}\right) = \frac{11{,}240 + 4630i}{877}$$

$$= \frac{11{,}240}{877} + \frac{4630}{877}i$$

61. False.

Sample answer: $(1 + i) + (3 + i) = 4 + 2i$ which is not a real number.

62. False.

If $b = 0$ then $a + bi = a - bi = a$.

That is, if the complex number is real, the number equals its conjugate.

63. True.

$$x^4 - x^2 + 14 = 56$$

$$\left(-i\sqrt{6}\right)^4 - \left(-i\sqrt{6}\right)^2 + 14 \stackrel{?}{=} 56$$

$$36 + 6 + 14 \stackrel{?}{=} 56$$

$$56 = 56$$

64. False.

$$i^{44} + i^{150} - i^{74} - i^{109} + i^{61} = \left(i^2\right)^{22} + \left(i^2\right)^{75} - \left(i^2\right)^{37} - \left(i^2\right)^{54}i + \left(i^2\right)^{30}i$$

$$= (-1)^{22} + (-1)^{75} - (-1)^{37} - (-1)^{54}i + (-1)^{30}i$$

$$= 1 - 1 + 1 - i + i = 1$$

65. $i = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i^4 i = i$

$i^6 = i^4 i^2 = -1$

$i^7 = i^4 i^3 = -i$

$i^8 = i^4 i^4 = 1$

$i^9 = i^4 i^4 i = i$

$i^{10} = i^4 i^4 i^2 = -1$

$i^{11} = i^4 i^4 i^3 = -i$

$i^{12} = i^4 i^4 i^4 = 1$

The pattern $i, -1, -i, 1$ repeats. Divide the exponent by 4.

If the remainder is 1, the result is i.

If the remainder is 2, the result is -1.

If the remainder is 3, the result is $-i$.

If the remainder is 0, the result is 1.

66. (a) $\left(-1+\sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(\sqrt{3}i\right) + 3(-1)\left(\sqrt{3}i\right)^2 + \left(\sqrt{3}i\right)^3$

$= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^3$

$= -1 + 3\sqrt{3}i - 9i^2 + 3\sqrt{3}i^2 i$

$= -1 + 3\sqrt{3}i + 9 - 3\sqrt{3}i$

$= 8$

(b) $\left(-1-\sqrt{3}i\right)^3 = (-1)^3 + 3(-1)^2\left(-\sqrt{3}i\right) + 3(-1)\left(-\sqrt{3}i\right)^2 + \left(-\sqrt{3}i\right)^3$

$= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^3$

$= -1 - 3\sqrt{3}i - 9i^2 - 3\sqrt{3}i^2 i$

$= -1 - 3\sqrt{3}i + 9 + 3\sqrt{3}i$

$= 8$

67. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = 6i^2 = -6$

68. (i) D
(ii) F
(iii) B
(iv) E
(v) A
(vi) C

69. $(a_1 + b_1 i)(a_2 + b_2 i) = a_1a_2 + a_1b_2 i + a_2b_1 i + b_1b_2 i^2$

$= (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1)i$

The complex conjugate of this product is
$(a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i.$

The product of the complex conjugates is

$(a_1 - b_1 i)(a_2 - b_2 i) = a_1a_2 - a_1b_2 i - a_2b_1 i - b_1b_2 i^2$

$= (a_1a_2 - b_1b_2) - (a_1b_2 + a_2b_1)i.$

So, the complex conjugate of the product of two complex numbers is the product of their complex conjugates.

70. $(a_1 + b_1 i) + (a_2 + b_2 i) = (a_1 + a_2) + (b_1 + b_2)i$

The complex conjugate of this sum is
$(a_1 + a_2) - (b_1 + b_2)i.$

The sum of the complex conjugates is
$(a_1 - b_1 i) + (a_2 - b_2 i) = (a_1 + a_2) - (b_1 + b_2)i.$

So, the complex conjugate of the sum of two complex numbers is the sum of their complex conjugates.

71. $3x^4 - 48x^2 = 3x^2(x^2 - 16) = 3x^2(x - 4)(x + 4)$

72. $9x^4 - 12x^2 = 3x^2(3x^2 - 4)$

73. $x^3 - 3x^2 + 3x - 9 = x^2(x - 3) + 3(x - 3)$

$= (x - 3)(x^2 + 3)$

74. $x^3 - 5x^2 - 2x + 10 = x^2(x - 5) - 2(x - 5)$

$= (x - 5)(x^2 - 2)$

75. $6x^3 - 27x^2 - 54x = 3x(2x^2 - 9x - 18)$

$= 3x(2x + 3)(x - 6)$

76. $12x^3 - 16x^2 - 60x = 4x(3x^2 - 4x - 15)$

$= 4x(3x + 5)(x - 3)$

77. $x^4 - 3x^2 + 2 = (x^2 - 1)(x^2 - 2)$

$= (x - 1)(x + 1)(x^2 - 2)$

78. $x^4 - 7x^2 + 12 = (x^2 - 3)(x^2 - 4)$

$= (x^2 - 3)(x - 2)(x + 2)$

79. $9x^4 - 37x^2 + 4 = (9x^2 - 1)(x^2 - 4)$

$= (3x + 1)(3x - 1)(x + 2)(x - 2)$

80. $4x^4 - 37x^2 + 9 = (4x^2 - 1)(x^2 - 9)$

$= (2x + 1)(2x - 1)(x + 3)(x - 3)$

81. (a) When $x = -3$, $\sqrt{2x + 7} - x$

$= \sqrt{2(-3) + 7} - (-3) = \sqrt{1} + 3 = 4.$

(b) When $x = 1$, $\sqrt{2x + 7} - x$

$= \sqrt{2(1) + 7} - 1 = \sqrt{9} - 1 = 2.$

82. (a) When $x = 4$, $x + \sqrt{40 - 9x} = 4 + \sqrt{40 - 9(4)} = 4 + \sqrt{4} = 4 + 2 = 6.$

(b) When $x = -9$, $x + \sqrt{40 - 9x} = -9 + \sqrt{40 - 9(-9)} = -9 + \sqrt{121} = -9 + 11 = 2.$

83. (a) When $x = 3$, $\sqrt{2x - 5} - \sqrt{x - 3} - 1 = \sqrt{2(3) - 5} - \sqrt{3 - 3} - 1 = \sqrt{1} - \sqrt{0} - 1 = 1 - 1 = 0.$

(b) When $x = 7$, $\sqrt{2x - 5} - \sqrt{x - 3} - 1 = \sqrt{2(7) - 5} - \sqrt{7 - 3} - 1 = \sqrt{9} - \sqrt{4} - 1 = 3 - 2 - 1 = 0.$

84. (a) When $x = 4$, $\sqrt{5x - 4} + \sqrt{x} - 1 = \sqrt{5(4) - 4} + \sqrt{4} - 1 = \sqrt{16} + \sqrt{4} - 1 = 5.$

(b) When $x = 1$, $\sqrt{5x - 4} + \sqrt{x} - 1 = \sqrt{5(1) - 4} + \sqrt{1} - 1 = \sqrt{1} + \sqrt{1} - 1 = 1.$

85. (a) When $x = 129$, $(x - 4)^{2/3} = (129 - 4)^{2/3} = 125^{2/3} = 25.$

(b) When $x = -121$, $(x - 4)^{2/3} = (-121 - 4)^{2/3} = (-125)^{2/3} = 25.$

86. (a) When $x = 69$, $(x - 5)^{3/2} = (69 - 5)^{3/2} = (64)^{3/2} = 8^3 = 512.$

(b) When $x = 14$, $(x - 5)^{3/2} = (14 - 5)^{3/2} = (9)^{3/2} = 3^3 = 27.$

87. (a) When $x = 4$, $\frac{2}{x} - \frac{3}{x - 2} + 1 = \frac{2}{4} - \frac{3}{4 - 2} + 1 = \frac{1}{2} - \frac{3}{2} + 1 = -1 + 1 = 0.$

(b) When $x = -1$, $\frac{2}{x} - \frac{3}{x - 2} + 1 = \frac{2}{-1} - \frac{3}{-1 - 2} + 1 = -2 + 1 + 1 = 0.$

88. (a) When $x = -4$, $\frac{4}{x} + \frac{2}{x + 3} + 3 = \frac{4}{-4} + \frac{2}{-4 + 3} + 3 = -1 - 2 + 3 = 0.$

(b) When $x = -1$, $\frac{4}{x} + \frac{2}{x + 3} + 3 = \frac{4}{-1} + \frac{2}{-1 + 3} + 3 = -4 + 1 + 3 = 0.$

89. (a) When $x = -3$, $|x^2 - 3x| + 4x - 6 = |(-3)^2 - 3(-3)| + 4(-3) - 6 = |9 + 9| - 12 - 6 = 0.$

(b) When $x = 2$, $|x^2 - 3x| + 4x - 6 = |(2)^2 - 3(2)| + 4(2) - 6 = |4 - 6| + 8 - 6 = 4.$

90. (a) When $x = -3$, $|x^2 + 4x| - 7x - 18 = |(-3)^2 + 4(-3)| - 7(-3) - 18 = |9 - 12| + 21 - 18 = 6.$

(b) When $x = -9$, $|x^2 + 4x| - 7x - 18 = |(-9)^2 + 4(-9)| - 7(-9) - 18 = |81 - 36| + 63 - 18 = |45| + 63 - 18$
$= 45 + 63 - 18 = 90$

Section 1.6 Other Types of Equations

1. polynomial

2. $. x(x - 3)$.

3. To eliminate or remove the radical from the equation $\sqrt{x + 2} = x$, square each side of the equation to produce the equation $x + 2 = x^2$.

4. The equation $x^4 - 2x + 4 = 0$ is *not* of quadratic type.

5.
$$6x^4 - 54x^2 = 0$$
$$6x^2(x^2 - 9) = 0$$
$$6x^2 = 0 \Rightarrow x = 0$$
$$x^2 - 9 = 0 \Rightarrow x = \pm 3$$

6.
$$36x^3 - 100x = 0$$
$$4x(9x^2 - 25) = 0$$
$$4x(3x + 5)(3x - 5) = 0$$
$$4x = 0 \Rightarrow x = 0$$
$$3x + 5 = 0 \Rightarrow x = -\tfrac{5}{3}$$
$$3x - 5 = 0 \Rightarrow x = \tfrac{5}{3}$$

7.
$$\begin{aligned} 5x^3 + 30x^2 + 45x &= 0 \\ 5x\left(x^2 + 6x + 9\right) &= 0 \\ 5x(x + 3)^2 &= 0 \\ 5x = 0 \Rightarrow x &= 0 \\ x + 3 = 0 \Rightarrow x &= -3 \end{aligned}$$

8.
$$\begin{aligned} 9x^4 - 24x^3 + 16x^2 &= 0 \\ x^2\left(9x^2 - 24x + 16\right) &= 0 \\ x^2(3x - 4)^2 &= 0 \\ x^2 = 0 \Rightarrow x &= 0 \\ 3x - 4 = 0 \Rightarrow x &= \tfrac{4}{3} \end{aligned}$$

9.
$$\begin{aligned} x^4 - 81 &= 0 \\ \left(x^2 + 9\right)(x + 3)(x - 3) &= 0 \\ x^2 + 9 = 0 \Rightarrow x &= \pm 3i \\ x + 3 = 0 \Rightarrow x &= -3 \\ x - 3 = 0 \Rightarrow x &= 3 \end{aligned}$$

10.
$$\begin{aligned} x^6 - 64 &= 0 \\ \left(x^3 - 8\right)\left(x^3 + 8\right) &= 0 \\ (x - 2)\left(x^2 + 2x + 4\right)(x + 2)\left(x^2 - 2x + 4\right) &= 0 \\ x - 2 = 0 \Rightarrow x &= 2 \\ x^2 + 2x + 4 = 0 \Rightarrow x &= -1 \pm \sqrt{3}i \\ x + 2 = 0 \Rightarrow x &= -2 \\ x^2 - 2x + 4 = 0 \Rightarrow x &= 1 \pm \sqrt{3}i \end{aligned}$$

11.
$$\begin{aligned} x^3 + 512 &= 0 \\ x^3 + 8^3 &= 0 \\ (x + 8)\left(x^2 - 8x + 64\right) &= 0 \\ x + 8 = 0 \Rightarrow x &= -8 \\ x^2 - 8x + 64 = 0 \Rightarrow x &= 4 \pm 4\sqrt{3}i \end{aligned}$$

12.
$$\begin{aligned} 27x^3 - 343 &= 0 \\ (3x)^3 - 7^3 &= 0 \\ (3x - 7)\left(9x^2 + 21x + 49\right) &= 0 \\ 3x - 7 = 0 \Rightarrow x &= \frac{7}{3} \\ 9x^2 + 21x + 49 = 0 &\Rightarrow -\frac{7}{6} \pm \frac{7\sqrt{3}}{6}i \end{aligned}$$

13.
$$\begin{aligned} x^3 + 2x^2 + 3x + 6 &= 0 \\ x^2(x + 2) + 3(x + 2) &= 0 \\ (x + 2)\left(x^2 + 3\right) &= 0 \\ x + 2 = 0 \Rightarrow x &= -2 \\ x^2 + 3 = 0 \Rightarrow x &= \pm\sqrt{3}i \end{aligned}$$

14.
$$\begin{aligned} x^4 + 2x^3 - 8x - 16 &= 0 \\ x^3(x + 2) - 8(x + 2) &= 0 \\ \left(x^3 - 8\right)(x + 2) &= 0 \\ (x - 2)\left(x^2 + 2x + 4\right)(x + 2) &= 0 \\ x - 2 = 0 \Rightarrow x &= 2 \\ x^2 + 2x + 4 = 0 \Rightarrow x &= -1 \pm \sqrt{3}i \\ x + 2 = 0 \Rightarrow x &= -2 \end{aligned}$$

15. $x^4 - 4x^2 + 3 = 0$

$(x^2)^2 - 4(x^2) + 3 = 0$

Let $u = x^2$.

$u^2 - 4u + 3 = 0$

$(u - 3)(u - 1) = 0$

$u - 3 = 0 \Rightarrow u = 3$

$u - 1 = 0 \Rightarrow u = 1$

$u = 1 \qquad u = 3$

$x^2 = 1 \qquad x^2 = 3$

$x = \pm 1 \qquad x = \pm\sqrt{3}$

16. $x^4 - 13x^2 + 36 = 0$

$(x^2)^2 - 13(x^2) + 36 = 0$

Let $u = x^2$.

$u^2 - 13u + 36 = 0$

$(u - 9)(u - 4) = 0$

$u - 9 = 0 \Rightarrow u = 9$

$u - 4 = 0 \Rightarrow u = 4$

$u = 9 \qquad u = 4$

$x^2 = 9 \qquad x^2 = 4$

$x = \pm 3 \qquad x = \pm 2$

17. $4x^4 - 65x^2 + 16 = 0$

$4(x^2) - 65(x^2) + 16 = 0$

Let $u = x^2$.

$4u - 65u + 16 = 0$

$(4u - 1)(u - 16) = 0$

$4u - 1 = 0 \Rightarrow u = \frac{1}{4}$

$u - 16 = 0 \Rightarrow u = 16$

$u = \frac{1}{4} \qquad u = 16$

$x^2 = \frac{1}{4} \qquad x^2 = 16$

$x = \pm\frac{1}{2} \qquad x = \pm 4$

18. $36t^4 + 29t^2 - 7 = 0$

$36(t^2)^2 + 29(t^2) - 7 = 0$

Let $u = t^2$.

$36u^2 + 29u - 7 = 0$

$(36u - 7)(u + 1) = 0$

$36u - 7 = 0 \Rightarrow u = \frac{7}{36}$

$u + 1 = 0 \Rightarrow u = -1$

$u = \dfrac{7}{36} \qquad u = -1$

$x^2 = \dfrac{7}{36} \qquad x^2 = -1$

$x = \pm\dfrac{\sqrt{7}}{6} \qquad x = \pm i$

19. $2x + 9\sqrt{x} = 5$

$2x + 9\sqrt{x} - 5 = 0$

$2(\sqrt{x})^2 + 9(\sqrt{x}) - 5 = 0$

Let $u = \sqrt{x}$.

$2u^2 + 9u - 5 = 0$

$(2u - 1)(u + 5) = 0$

$2u - 1 = 0 \Rightarrow u = \frac{1}{2}$

$u + 5 = 0 \Rightarrow u = -5$

$u = \frac{1}{2} \qquad u = -5 \Rightarrow \sqrt{x} \neq -5$

$\sqrt{x} = \frac{1}{2} \qquad (\sqrt{x} = -5 \text{ is not a solution.})$

$x = \frac{1}{4}$

20. $6x - 7\sqrt{x} - 3 = 0$

$6\left(\sqrt{x}\right)^2 - 7\left(\sqrt{x}\right) - 3 = 0$

Let $u = \sqrt{x}$.

$6u^2 - 7u - 3 = 0$

$(3u + 1)(2u - 2) = 0$

$3u + 1 = 0 \Rightarrow u = -\frac{1}{3}$

$2u - 3 = 0 \Rightarrow u = \frac{3}{2}$

$u = -\frac{1}{3}$ $\qquad$ $u = \frac{3}{2}$

$\sqrt{x} \neq -\frac{1}{3}$ $\qquad$ $\sqrt{x} = \frac{3}{2}$

$\left(\sqrt{x} = -\frac{1}{3}\text{ is not a solution.}\right)$

21. $9t^{2/3} + 24t^{1/3} + 16 = 0$

$9\left(t^{1/3}\right)^2 + 24\left(t^{1/3}\right) + 16 = 0$

Let $u = t^{1/3}$.

$9u^2 + 24u + 16 = 0$

$(3u + 4)^2 = 0$

$3u + 4 = 0 \Rightarrow u = -\frac{4}{3}$

$u = -\frac{4}{3}$

$t^{1/3} = -\frac{4}{3}$

$t = -\frac{64}{27}$

22. $3x^{1/3} + 2x^{2/3} = 5$

$2x^{2/3} + 3x^{1/3} - 5 = 0$

$2\left(x^{1/3}\right)^2 + 3\left(x^{1/3}\right) - 5 = 0$

Let $u = x^{1/3}$.

$(2u + 5)(u - 1) = 0$

$2u + 5 = 0 \Rightarrow u = -\frac{5}{2}$

$u - 1 = 0 \Rightarrow u = 1$

$u = -\frac{5}{2}$ $\qquad$ $u = 1$

$x^{1/3} = -\frac{5}{2}$ $\qquad$ $x^{1/3} = 1$

$x = -\frac{125}{2}$ $\qquad$ $x = 1$

23. $\frac{1}{x^2} + \frac{8}{x} + 15 = 0$

$\left(\frac{1}{x}\right)^2 + 8\left(\frac{1}{x}\right) + 15 = 0$

Let $u = \frac{1}{x}$.

$u^2 + 8u + 15 = 0$

$(u + 5)(u + 3) = 0$

$u + 5 = 0 \Rightarrow u = -5$

$u + 3 = 0 \Rightarrow u = -3$

$u = -5$ $\qquad$ $u = -3$

$\frac{1}{x} = -5$ $\qquad$ $\frac{1}{x} = -3$

$x = -\frac{1}{5}$ $\qquad$ $x = -\frac{1}{3}$

24. $1 + \frac{3}{x} = -\frac{2}{x^2}$

$\frac{2}{x^2} + \frac{3}{x} + 1 = 0$

$2\left(\frac{1}{x}\right)^2 + 3\left(\frac{1}{x}\right) + 1 = 0$

Let $u = \frac{1}{x}$.

$2u^2 + 3u + 1 = 0$

$(2u + 1)(u + 1) = 0$

$2u + 1 = 0 \Rightarrow u = -\frac{1}{2}$

$u + 1 = 0 \Rightarrow u = -1$

$u = -\frac{1}{2}$ $\qquad$ $u = -1$

$\frac{1}{x} = -\frac{1}{2}$ $\qquad$ $\frac{1}{x} = -1$

$x = -2$ $\qquad$ $x = -1$

25. $2\left(\frac{x}{x+2}\right)^2 - 3\left(\frac{x}{x+2}\right) - 2 = 0$

Let $u = \frac{x}{x+2}$.

$2u^2 - 3u - 2 = 0$

$(2u+1)(u-2) = 0$

$2u + 1 = 0 \Rightarrow u = -\frac{1}{2}$

$u - 2 = 0 \Rightarrow u = 2$

$u = -\frac{1}{2}$ $\qquad$ $u = 2$

$\frac{x}{x+2} = -\frac{1}{2}$ $\qquad$ $\frac{x}{x+2} = 2$

$x = -\frac{2}{3}$ $\qquad$ $x = -4$

26. $6\left(\frac{x}{x+1}\right)^2 + 5\left(\frac{x}{x+1}\right) - 6 = 0$

Let $u = \frac{x}{x+1}$.

$6u^2 + 5u - 6 = 0$

$(3u-2)(2u+3) = 0$

$3u - 2 = 0 \Rightarrow u = \frac{2}{3}$

$2u + 3 = 0 \Rightarrow u = -\frac{3}{2}$

$u = \frac{2}{3}$ $\qquad$ $u = -\frac{3}{2}$

$\frac{x}{x+1} = \frac{2}{3}$ $\qquad$ $\frac{x}{x+1} = -\frac{3}{2}$

$x = 2$ $\qquad$ $x = -\frac{3}{5}$

27. $\sqrt{5x} - 10 = 0$

$\sqrt{5x} = 10$

$\left(\sqrt{5x}\right)^2 = (10)^2$

$5x = 100$

$x = 20$

28. $\sqrt{3x+1} = 7$

$\left(\sqrt{3x+1}\right)^2 = (7)^2$

$3x + 1 = 49$

$3x = 48$

$x = 16$

29. $4 + \sqrt[3]{2x-9} = 0$

$\sqrt[3]{2x-9} = -4$

$\left(\sqrt[3]{2x-9}\right)^3 = (-4)^3$

$2x - 9 = -64$

$2x = -55$

$x = -\frac{55}{2}$

30. $\sqrt[3]{12-x} - 3 = 0$

$\sqrt[3]{12-x} = 3$

$\left(\sqrt[3]{12-x}\right)^3 = (3)^3$

$12 - x = 27$

$-x = 15$

$x = -15$

31. $\sqrt{x+8} = 2 + x$

$\left(\sqrt{x+8}\right)^2 = (2+x)^2$

$x + 8 = x^2 + 4x + 4$

$0 = x^2 + 3x - 4$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x + 4 = 0 \Rightarrow x = -4$, extraneous

$x - 1 = 0 \Rightarrow x = 1$

32. $2x = \sqrt{-5x+24} - 3$

$2x + 3 = \sqrt{-5x+24}$

$(2x+3)^2 = \left(\sqrt{-5x+24}\right)^2$

$4x^2 + 12x + 9 = -5x + 24$

$4x^2 + 17x - 15 = 0$

$(4x-3)(x+5) = 0$

$4x - 3 = 0 \Rightarrow x = \frac{3}{4}$

$x + 5 \Rightarrow x = -5$, extraneous

33. $\sqrt{x-3} + 1 = \sqrt{x}$

$\sqrt{x-3} = \sqrt{x} - 1$

$\left(\sqrt{x-3}\right)^2 = \left(\sqrt{x} - 1\right)^2$

$x - 3 = x - 2\sqrt{x} + 1$

$-4 = -2\sqrt{x}$

$2 = \sqrt{x}$

$(2)^2 = \left(\sqrt{x}\right)^2$

$4 = x$

34. $\sqrt{x} + \sqrt{x - 24} = 2$

$$\begin{aligned}
\sqrt{x} &= 2 - \sqrt{x - 24}\\
\left(\sqrt{x}\right)^2 &= \left(2 - \sqrt{x - 24}\right)^2\\
x &= 4 - 4\sqrt{x - 24} + x - 24\\
20 &= -4\sqrt{x - 24}\\
5 &= -\sqrt{x - 24}\\
5^2 &= \left(-\sqrt{x - 24}\right)^2\\
25 &= x - 24\\
49 &= x
\end{aligned}$$

$x = 49$ is an extraneous solution, so the equation has no solution.

35. $2\sqrt{x + 1} - \sqrt{2x + 3} = 1$

$$\begin{aligned}
2\sqrt{x + 1} &= 1 + \sqrt{2x + 3}\\
\left(2\sqrt{x + 1}\right)^2 &= \left(1 + \sqrt{2x + 3}\right)^2\\
4(x + 1) &= 1 + 2\sqrt{2x + 3} + 2x + 3\\
2x &= 2\sqrt{2x + 3}\\
x &= \sqrt{2x + 3}\\
x^2 &= 2x + 3\\
x^2 - 2x - 3 &= 0\\
(x - 3)(x + 1) &= 0\\
x - 3 &= 0 \Rightarrow x = 3\\
x + 1 &= 0 \Rightarrow x = -1, \text{ extraneous}
\end{aligned}$$

36. $4\sqrt{x - 3} - \sqrt{6x - 17} = 3$

$$\begin{aligned}
4\sqrt{x - 3} &= 3 + \sqrt{6x - 17}\\
\left(4\sqrt{x - 3}\right)^2 &= \left(3 + \sqrt{6x - 17}\right)^2\\
16(x - 3) &= 9 + 6\sqrt{6x - 17} + 6x - 17\\
16x - 48 &= 6\sqrt{6x - 17} + 6x - 8\\
10x - 40 &= 6\sqrt{6x - 17}\\
5x - 20 &= 3\sqrt{6x - 17}\\
(5x - 20)^2 &= \left(3\sqrt{6x - 17}\right)^2\\
25x^2 - 200x + 400 &= 9(6x - 17)\\
25x^2 - 200x + 400 &= 54x - 153\\
25x^2 - 254x + 553 &= 0\\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\
x &= \frac{-(-254) \pm \sqrt{(-254)^2 - 4(25)(553)}}{2(25)}\\
x &= \frac{254 \pm \sqrt{9261}}{50}\\
x &= \frac{254 + 96}{50} = \frac{350}{50} = 7\\
x &= \frac{254 - 96}{50} = \frac{158}{50} = \frac{79}{25}, \text{ extraneous}
\end{aligned}$$

37. $\sqrt{4\sqrt{4x+9}} = \sqrt{8x+2}$

$\left(\sqrt{4\sqrt{4x+9}}\right)^2 = \left(\sqrt{8x+2}\right)^2$

$4\sqrt{4x+9} = 8x+2$

$2\sqrt{4x+9} = 4x+1$

$\left(2\sqrt{4x+9}\right)^2 = (4x+1)^2$

$4(4x+9) = 16x^2 + 8x + 1$

$16x + 36 = 16x^2 + 8x + 1$

$0 = 16x^2 - 8x - 35$

$0 = (4x+5)(4x-7)$

$4x + 5 = 0 \Rightarrow x = -\frac{5}{4}$, extraneous

$4x - 7 = 0 \Rightarrow x = \frac{7}{4}$

38. $\sqrt{16 + 9\sqrt{x}} = 4 + \sqrt{x}$

$\left(\sqrt{16+9\sqrt{x}}\right)^2 = \left(4+\sqrt{x}\right)^2$

$16 + 9\sqrt{x} = 16 + 8\sqrt{x} + x$

$\sqrt{x} = x$

$\left(\sqrt{x}\right)^2 = (x)^2$

$x = x^2$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0$

$x - 1 = 0 \Rightarrow x = 1$

39. $(x-5)^{3/2} = 8$

$(x-5)^3 = 8^2$

$x - 5 = \sqrt[3]{64}$

$x = 5 + 4 = 9$

40. $(x+2)^{2/3} = 9$

$(x+2)^2 = 9^3$

$x + 2 = \pm\sqrt{729}$

$x = -2 \pm 27 = -29, 25$

41. $(x^2-5)^{3/2} = 27$

$(x^2-5)^3 = 27^2$

$x^2 - 5 = \sqrt[3]{27^2}$

$x^2 = 5 + 9$

$x^2 = 14$

$x = \pm\sqrt{14}$

42. $(x^2 - x - 22)^{3/2} = 27$

$x^2 - x - 22 = 27^{2/3}$

$x^2 - x - 22 = 9$

$x^2 - x - 31 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-31)}}{2(1)} = \frac{1 \pm \sqrt{125}}{2} = \frac{1 \pm 5\sqrt{5}}{2}$$

43. $3x(x-1)^{1/2} + 2(x-1)^{3/2} = 0$

$(x-1)^{1/2}\left[3x + 2(x-1)\right] = 0$

$(x-1)^{1/2}(5x-2) = 0$

$(x-1)^{1/2} = 0 \Rightarrow x - 1 = 0 \Rightarrow x = 1$

$5x - 2 = 0 \Rightarrow x = \frac{2}{5}$, extraneous

44.
$$\begin{aligned}4x^2(x-1)^{1/3} + 6x(x-1)^{4/3} &= 0\\ 2x\left[2x(x-1)^{1/3} + 3(x-1)^{4/3}\right] &= 0\\ 2x(x-1)^{1/3}\left[2x + 3(x-1)\right] &= 0\\ 2x(x-1)^{1/3}(5x-3) &= 0\\ 2x = 0 \Rightarrow x &= 0\\ x-1 = 0 \Rightarrow x &= 1\\ 5x-3 = 0 \Rightarrow x &= \tfrac{3}{5}\end{aligned}$$

45.
$$\begin{aligned}\frac{1}{x} - \frac{1}{x+1} &= 3\\ x(x+1)\frac{1}{x} - x(x+1)\frac{1}{x+1} &= x(x+1)(3)\\ x+1-x &= 3x(x+1)\\ 1 &= 3x^2+3x\\ 0 &= 3x^2+3x-1\end{aligned}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(3)(-1)}}{2(3)} = \frac{-3 \pm \sqrt{21}}{6}$$

46.
$$\begin{aligned}\frac{4}{x+1} - \frac{3}{x+2} &= 1\\ 4(x+2) - 3(x+1) &= (x+1)(x+2)\\ 4x+8-3x &= x^2+3x+2\\ x^2+2x-3 &= 0\\ (x-1)(x+3) &= 0\\ x-1 = 0 \Rightarrow x &= 1\\ x+3 = 0 \Rightarrow x &= -3\end{aligned}$$

47.
$$\begin{aligned}3 - \frac{14}{x} - \frac{5}{x^2} &= 0\\ \frac{5}{x^2} + \frac{14}{x} - 3 &= 0\\ 5\left(\frac{1}{x}\right)^2 + 14\left(\frac{1}{x}\right) - 3 &= 0\end{aligned}$$

Let $u = \frac{1}{x}$.

$$\begin{aligned}5u^2+14u-3 &= 0\\ (5u-1)(u+3) &= 0\\ 5u-1 = 0 \Rightarrow u &= \frac{1}{5}\\ u+3 = 0 \Rightarrow u &= -3\end{aligned}$$

$u = \frac{1}{5}$ $u = -3$

$\frac{1}{x} = \frac{1}{5}$ $\frac{1}{x} = -3$

$x = 5$ $x = -\frac{1}{3}$

48.
$$\begin{aligned}5 &= \frac{18}{x} + \frac{8}{x^2}\\ \frac{8}{x^2} + \frac{18}{x} - 5 &= 0\\ 8\left(\frac{1}{x}\right)^2 + 18\left(\frac{1}{x}\right) - 5 &= 0\end{aligned}$$

Let $u = \frac{1}{x}$.

$$\begin{aligned}8u^2+18u-5 &= 0\\ (4u-1)(2u+5) &= 0\\ 4u-1 = 0 \Rightarrow u &= \frac{1}{4}\\ 2u+5 = 0 \Rightarrow u &= -\frac{5}{2}\end{aligned}$$

$u = \frac{1}{4}$ $u = -\frac{5}{2}$

$\frac{1}{x} = \frac{1}{4}$ $\frac{1}{x} = -\frac{5}{2}$

$x = 4$ $x = -\frac{2}{5}$

49.
$$\begin{aligned}\frac{x+1}{3} - \frac{x+1}{x+2} &= 0\\ 3(x+2)\frac{x+1}{3} - 3(x+2)\frac{x+1}{x+2} &= 0\\ (x+2)(x+1) - 3(x+1) &= 0\\ x^2+3x+2-3x-3 &= 0\\ x^2-1 &= 0\\ (x+1)(x-1) &= 0\\ x+1 = 0 \Rightarrow x &= -1\\ x-1 = 0 \Rightarrow x &= 1\end{aligned}$$

50.

$$\frac{x}{x^2 - 4} + \frac{1}{x + 2} = 3$$

$$(x + 2)(x - 2)\frac{x}{x^2 - 4} + (x + 2)(x - 2)\frac{1}{x + 2} = 3(x + 2)(x - 2)$$

$$x + x - 2 = 3x^2 - 12$$

$$3x^2 - 2x - 10 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-10)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{124}}{6} = \frac{2 \pm 2\sqrt{31}}{6} = \frac{1 \pm \sqrt{31}}{3}$$

51. $|2x - 5| = 11$

$$2x - 5 = 11 \Rightarrow x = 8$$

$$-(2x - 5) = 11 \Rightarrow x = -3$$

52. $|3x + 2| = 7$

$$3x + 2 = 7 \Rightarrow x = \tfrac{5}{3}$$

$$-(3x + 2) = 7$$

$$-3x - 2 = 7 \Rightarrow x = -3$$

53. $|x| = x^2 + x - 24$

First equation:

$$x = x^2 + x - 24$$

$$x^2 - 24 = 0$$

$$x^2 = 24$$

$$x = \pm 2\sqrt{6}$$

Second equation:

$$-x = x^2 + x - 24$$

$$x^2 + 2x - 24 = 0$$

$$(x + 6)(x - 4) = 0$$

$$x + 6 = 0 \Rightarrow x = -6$$

$$x - 4 = 0 \Rightarrow x = 4$$

Only $x = 2\sqrt{6}$ and $x = -6$ are solutions of the original equation. $x = -2\sqrt{6}$ and $x = 4$ are extraneous.

54. $|x^2 + 6x| = 3x + 18$

First equation:

$$x^2 + 6x = 3x + 18$$

$$x^2 + 3x - 18 = 0$$

$$(x - 3)(x + 6) = 0$$

$$x - 3 = 0 \Rightarrow x = 3$$

$$x + 6 = 0 \Rightarrow x = -6$$

Second equation:

$$-(x^2 + 6x) = 3x + 18$$

$$0 = x^2 + 9x + 18$$

$$0 = (x + 3)(x + 6)$$

$$0 = x + 3 \Rightarrow x = -3$$

$$x = x + 6 \Rightarrow x = -6$$

The solutions of the original equation are $x = \pm 3$ and $x = -6$.

55. $|x + 1| = x^2 - 5$

First equation:

$$x + 1 = x^2 - 5$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x - 3 = 0 \Rightarrow x = 3$$

$$x + 2 = 0 \Rightarrow x = -2$$

Second equation:

$$-(x + 1) = x^2 - 5$$

$$-x - 1 = x^2 - 5$$

$$x^2 + x - 4 = 0$$

$$x = \frac{-1 + \sqrt{17}}{2}$$

Only $x = 3$ and $x = \frac{-1 - \sqrt{17}}{2}$ are solutions of the original equation. $x = -2$ and $x = \frac{-1 + \sqrt{17}}{2}$ are extraneous.

56. $|x - 15| = x^2 - 15x$

First equation:

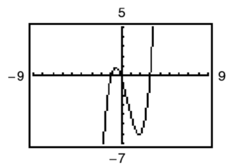
$$x - 15 = x^2 - 15x$$
$$x^2 - 16x + 15 = 0$$
$$(x - 1)(x - 15) = 0$$
$$x - 1 = 0 \Rightarrow x = 1$$
$$x - 15 = 0 \Rightarrow x = 15$$

Second equation:

$$-(x - 15) = x^2 - 15x$$
$$x^2 - 14x - 15 = 0$$
$$(x + 1)(x - 15) = 0$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x - 15 = 0 \Rightarrow x = 15$$

Only $x = 15$ and $x = -1$ are solutions of the original equation. $x = 1$ is extraneous.

57. (a)

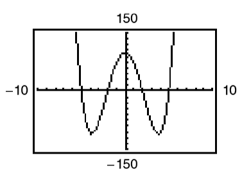


(b) x-intercepts: $(-1, 0), (0, 0), (3, 0)$

(c)
$$0 = x^3 - 2x^2 - 3x$$
$$0 = x(x + 1)(x - 3)$$
$$x = 0$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x - 3 = 0 \Rightarrow x = 3$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

58. (a)



(b) x-intercepts: $(-2, 0), (2, 0), (-5, 0), (5, 0)$

(c)
$$y = x^4 - 29x^2 + 100$$
$$0 = x^4 - 29x^2 + 100$$
$$0 = (x^2)^2 - 29(x^2) + 100$$

Let $u = x^2$.

$$0 = u^2 - 29u + 100$$
$$0 = (u - 4)(u - 25)$$

$u - 4 = 0 \qquad u - 25 = 0$

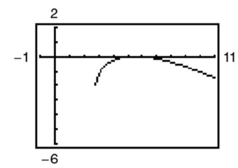
$u = 4 \qquad u = 25$

$x^2 = 4 \qquad x^2 = 25$

$x = \pm 2 \qquad x = \pm 5$

(d) The x-intercepts and the solutions are the same.

59. (a)

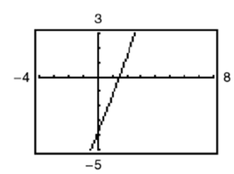


(b) x-intercepts: $(5, 0), (6, 0)$

(c)
$$0 = \sqrt{11x - 30} - x$$
$$x = \sqrt{11x - 30}$$
$$x^2 = 11x - 30$$
$$x^2 - 11x + 30 = 0$$
$$(x - 5)(x - 6) = 0$$
$$x - 5 = 0 \Rightarrow x = 5$$
$$x - 6 = 0 \Rightarrow x = 6$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

60. (a)

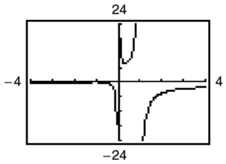


(b) x-intercept: $\left(\frac{3}{2}, 0\right)$

(c)
$$y = 2x - \sqrt{15 - 4x}$$
$$0 = 2x - \sqrt{15 - 4x}$$
$$\sqrt{15 - 4x} = 2x$$
$$15 - 4x = 4x^2$$
$$0 = 4x^2 + 4x - 15$$
$$0 = (2x + 5)(2x - 3)$$
$$0 = 2x + 5 \Rightarrow x = -\tfrac{5}{2}, \text{ extraneous}$$
$$0 = 2x - 3 \Rightarrow x = \tfrac{3}{2}$$
$$x = \tfrac{3}{2}$$

(d) The x-intercept and the solution are the same.

61. (a)

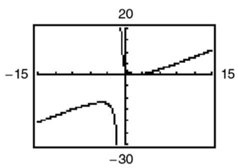


(b) x-intercept: $(-1, 0)$

(c)
$$0 = \frac{1}{x} - \frac{4}{x-1} - 1$$
$$0 = (x-1) - 4x - x(x-1)$$
$$0 = x - 1 - 4x - x^2 + x$$
$$0 = -x^2 - 2x - 1$$
$$0 = x^2 + 2x + 1$$
$$0 = (x+1)^2$$
$$x + 1 = 0 \Rightarrow x = -1$$

(d) The x-intercept of the graph is the same as the solution of the equation.

62. (a)

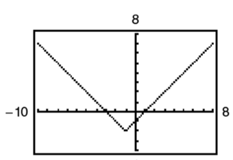


(b) x-intercept: $(2, 0)$

(c)
$$0 = x + \frac{9}{x+1} - 5$$
$$0 = x + \frac{9}{x+1} - 5$$
$$0 = x(x+1) + (x+1)\frac{9}{x+1} - 5(x+1)$$
$$0 = x^2 + x + 9 - 5x - 5$$
$$0 = x^2 - 4x + 4$$
$$0 = (x-2)(x-2)$$
$$0 = x - 2 \Rightarrow x = 2$$
$$x = 2$$

(d) The x-intercept and the solution are the same.

63. (a)

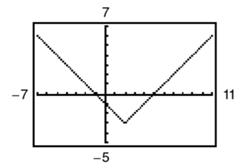


(b) x-intercepts: $(1, 0), (-3, 0)$

(c)
$$0 = |x+1| - 2$$
$$2 = |x+1|$$
$$x + 1 = 2 \quad \text{or} \quad -(x+1) = 2$$
$$x = 1 \quad \text{or} \quad -x - 1 = 2$$
$$-x = 3$$
$$x = -3$$

(d) The x-intercepts of the graph are the same as the solutions of the equation.

64. (a)



(b) x-intercepts: $(5, 0), (-1, 0)$

(c)
$$0 = |x-2| - 3$$
$$3 = |x-2|$$
$$x - 2 = 3 \Rightarrow x = 5 \quad \text{or} \quad -(x-2) = 3$$
$$-x + 2 = 3 \Rightarrow x = -1$$
$$x = 5, -1$$

(d) The x-intercepts and the solutions are the same.

65.
$$x^3 - 3x^2 - 1.21x + 3.63 = 0$$
$$x^2(x-3) - 1.21(x-3) = 0$$
$$(x-3)(x^2 - 1.21) = 0$$
$$(x-3)(x+1.1)(x-1.1) = 0$$
$$x = 3, \pm 1.1$$

66.
$$x^4 - 1.7x^3 + x = 1.7$$
$$x^4 - 1.7x^3 + x - 1.7 = 0$$
$$x^3(x-1.7) + (x-1.7) = 0$$
$$(x^3+1)(x-1.7) = 0$$
$$(x+1)(x^2 - x + 1)(x-1.7) = 0$$
$$x = -1, 1.7$$

67. $3.2x^4 - 1.5x^2 - 2.1 = 0$

$$x^2 = \frac{1.5 \pm \sqrt{1.5^2 - 4(3.2)(-2.1)}}{2(3.2)}$$

Using the positive value for x^2, we have

$$x = \pm\sqrt{\frac{1.5 + \sqrt{29.13}}{6.4}} \approx \pm 1.038.$$

68. $0.1x^4 - 2.4x^2 - 3.6 = 0$

$$x^2 = \frac{2.4 \pm \sqrt{(-2.4)^2 - 4(0.1)(-3.6)}}{2(0.1)} = \frac{2.4 \pm 7.2}{0.2}$$

Using the positive values for x^2,

$$x = \pm\sqrt{\frac{2.4 + \sqrt{7.2}}{0.2}} \approx \pm 5.041.$$

69. $7.08x^6 + 4.15x^3 - 9.6 = 0$

$a = 7.8,\ b = 4.15,\ c = -9.6$

$$x^3 = \frac{-4.15 \pm \sqrt{(4.15)^2 - 4(7.08)(-9.6)}}{2(7.08)}$$

$$= \frac{-4.15 \pm \sqrt{2.89.0945}}{14.16}$$

$$x = \sqrt[3]{\frac{-4.15 + \sqrt{289.0945}}{14.16}} \approx 0.968$$

$$x = \sqrt[3]{\frac{-4.15 - \sqrt{289.0945}}{14.16}} \approx -1.143$$

70. $5.25x^6 - 0.2x^3 - 1.55 = 0$

$$x^3 = \frac{0.2 \pm \sqrt{(-0.2)^2 - 4(5.25)(-1.55)}}{2(5.25)}$$

$$= \frac{0.2 \pm \sqrt{32.59}}{10.5}$$

$$x = \sqrt[3]{\frac{0.2 + \sqrt{32.59}}{10.5}} \approx 0.826$$

$$x = \sqrt[3]{\frac{0.2 - \sqrt{32.59}}{10.5}} \approx -0.807$$

71. $11.5 - 5.6\sqrt{x} = 0$

$$-5.6\sqrt{x} = -11.5$$

$$\sqrt{x} = \frac{-11.5}{-5.6}$$

$$x = \left(\frac{11.5}{5.6}\right)^2 \approx 4.217$$

72. $\sqrt{x + 8.2} - 5.55 = 0$

$$\sqrt{x + 8.2} = 5.55$$

$$x + 8.2 = (5.55)^2$$

$$x = (5.55)^2 - 8.2 \approx 22.603$$

73. $1.8x - 6\sqrt{x} - 5.6 = 0$ Given equation

$$1.8\left(\sqrt{x}\right)^2 - 6\sqrt{x} - 5.6 = 0$$

Use the Quadratic Formula with $a = 1.8,\ b = -6,$ and $c = -5.6.$

$$\sqrt{x} = \frac{6 \pm \sqrt{36 - 4(1.8)(-5.6)}}{2(1.8)} \approx \frac{6 \pm 8.7361}{3.6}$$

Considering only the positive value for $\sqrt{x}$, we have

$$\sqrt{x} \approx 4.0934$$

$$x \approx 16.756.$$

74. $5.3x + 3.1 = 9.8\sqrt{x}$

$$(5.3x + 3.1)^2 = \left(9.8\sqrt{x}\right)^2$$

$$28.09x^2 + 32.86x + 9.61 = 96.04x$$

$$28.09x^2 - 63.18x + 9.61 = 0$$

$$x = \frac{-(-63.18) \pm \sqrt{(-63.18)^2 - 4(28.09)(9.61)}}{2(28.09)}$$

$$= \frac{63.18 \pm \sqrt{2911.9328}}{56.18}$$

$$\approx 2.085,\ 0.164$$

75. $4x^{2/3} + 8x^{1/3} + 3.6 = 0$

$a = 4,\ b = 8,\ c = 3.6$

$$x^{1/3} = \frac{-8 \pm \sqrt{8^2 - 4(4)(3.6)}}{2(4)}$$

$$x = \left[\frac{-8 + \sqrt{6.4}}{8}\right]^3 \approx -0.320$$

$$x = \left[\frac{-8 - \sqrt{6.4}}{8}\right]^3 \approx -2.280$$

76. $8.4x^{2/3} - 1.2x^{1/3} - 24 = 0$

$$x^{1/3} = \frac{1.2 \pm \sqrt{(-1.2)^2 - 4(8.4)(-24)}}{2(8.4)}$$

$$= \frac{1.2 \pm \sqrt{807.84}}{16.8}$$

$$x = \left(\frac{1.2 + \sqrt{807.84}}{16.8}\right)^3 \approx 5.482$$

$$x = \left(\frac{1.2 - \sqrt{807.84}}{16.8}\right)^3 \approx -4.255$$

$$x = \frac{3.3}{x} + \frac{1}{2.2}$$

$$x^2 = 3.3 + \frac{5}{11}x$$

77. $x^2 - \frac{5}{11}x - 3.3 = 0$

$$x^2 = \frac{(5/11) \pm \sqrt{(5/11)^2 - 4(1)(-3.3)}}{2(1)}$$

$$x \approx -1.603,\ 2.058$$

78. $\frac{4.4}{x} - \frac{5.5}{3.3} = \frac{x}{6.6}$

$$4.4 - \frac{5}{3}x = \frac{5}{33}x^2$$

$$0 = \frac{5}{33}x^2 + \frac{5}{3}x - 4.4$$

$$x = \frac{(-5/3) \pm \sqrt{(5/3)^2 - 4(5/33)(-4.4)}}{2(5/33)}$$

$$x = -13.2,\ 2.2$$

79. $-4, 7$

Sample answer: $(x - (-4))(x - 7) = 0$

$$(x + 4)(x - 7) = 0$$

$$x^2 - 3x - 28 = 0$$

80. $0, 2, 9$

Sample answer: $(x - 0)(x - 2)(x - 9) = 0$

$$x(x - 2)(x - 9) = 0$$

$$x(x^2 - 11x + 18) = 0$$

$$x^3 - 11x^2 + 18x = 0$$

81. $-\frac{7}{3}, \frac{6}{7}$

One possible equation is:

$x = -\frac{7}{3} \Rightarrow 3x = -7 \Rightarrow 3x + 7$ is a factor.

$x = \frac{6}{7} \Rightarrow 7x = 6 \Rightarrow 7x - 6$ is a factor.

$$(3x + 7)(7x - 6) = 0$$

$$21x^2 + 31x - 42 = 0$$

Any non-zero multiple of this equation would also have these solutions.

82. $-\frac{1}{8}, -\frac{4}{5}$

$$\left(x - \left(-\tfrac{1}{8}\right)\right)\left(x - \left(-\tfrac{4}{5}\right)\right) = 0$$

$$\left(x + \tfrac{1}{8}\right)\left(x + \tfrac{4}{5}\right) = 0$$

$$x^2 + \tfrac{4}{5}x + \tfrac{1}{8}x + \tfrac{4}{40} = 0$$

$$40x^2 + 32x + 5x + 4 = 0$$

$$40x^2 + 37x + 4 = 0$$

Any non-zero multiple of this equation would also have these solutions.

83. $\sqrt{3}, -\sqrt{3}$, and 4

One possible equation is:

$$\left(x - \sqrt{3}\right)\left(x - \left(-\sqrt{3}\right)\right)(x - 4) = 0$$

$$\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)(x - 4) = 0$$

$$(x^2 - 3)(x - 4) = 0$$

$$x^3 - 4x^2 - 3x + 12 = 0$$

Any non-zero multiple of this equation would also have these solutions.

84. $2\sqrt{7}, -\sqrt{7}$

$$\left(x - 2\sqrt{7}\right)\left(x + \sqrt{7}\right) = 0$$

$$x^2 + x\sqrt{7} - 2x\sqrt{7} - 2(7) = 0$$

$$x^2 - x\sqrt{7} - 14 = 0$$

Any non-zero multiple of this equation would also have these solutions.

85. $i, -i$

Sample answer: $(x - i)(x - (-i)) = 0$

$$(x - i)(x + i) = 0$$

$$x^2 - i^2 = 0$$

$$x^2 + 1 = 0$$

86. $2i, -2i$

Sample answer: $(x - 2i)(x - (-2i)) = 0$

$$(x - 2i)(x + 2i) = 0$$

$$x^2 - 4i^2 = 0$$

$$x^2 + 4 = 0$$

87. $-1, 1, i$, and $-i$

One possible equation is:

$$(x - (-1))(x - 1)(x - i)(x - (-i)) = 0$$

$$(x + 1)(x - 1)(x - i)(x + i) = 0$$

$$(x^2 - 1)(x^2 + 1) = 0$$

$$x^4 - 1 = 0$$

Any non-zero multiple of this equation would also have these solutions.

88. $4i, -4i, 6, -6$

Sample answer: $(x - 4i)(x + 4i)(x - 6)(x + 6) = 0$

$$(x^2 + 16)(x^2 - 36) = 0$$

$$x^4 - 20x^2 - 576 = 0$$

89. *Labels*: Let $x =$ the number of students in the original group. Then, $\dfrac{1700}{x} =$ the original cost per student.

When six more students join the group, the cost per student becomes $\dfrac{1700}{x} - 7.50$.

Model: (Cost per student) · (Number of students) = (Total cost)

Equation: $\left(\dfrac{1700}{x} - 7.5\right)(x + 6) = 1700$

$(3400 - 15x)(x + 6) = 3400x$ Multiply both sides by $2x$ to clear fraction.

$$-15x^2 - 90x + 20{,}400 = 0$$

$$x = \frac{90 \pm \sqrt{(-90)^2 - 4(-15)(20{,}400)}}{2(-15)} = \frac{90 \pm 1110}{-30}$$

Using the positive value for x, we conclude that the original number was $x = 34$ students.

90. *Model:* $\begin{pmatrix}\text{Cost per}\\\text{student}\end{pmatrix} \cdot \begin{pmatrix}\text{Number of}\\\text{students}\end{pmatrix} = \begin{pmatrix}\text{Monthly}\\\text{rent}\end{pmatrix}$

Labels: Monthly rent $= x$

Number of students $= 4$

Original cost per student $= \dfrac{x}{3}$

Cost per student $= \dfrac{x}{3} - 150$

Equation: $\left(\dfrac{x}{3} - 150\right)(4) = x$

$$\frac{4x}{3} - 600 = x$$

$$\frac{4x}{3} - x = 600$$

$$\frac{x}{3} = 600$$

$$x = 1800$$

The monthly rent is \$1800.

91. *Model:* $\text{Time} = \dfrac{\text{Distance}}{\text{Rate}}$

Labels: Let $x =$ average speed of the plane. Then we have a travel time of $t = 145/x$. If the average speed is increased by 40 mph, then

$$t - \frac{12}{60} = \frac{145}{x + 40}$$

$$t = \frac{145}{x + 40} + \frac{1}{5}.$$

Now, we equate these two equations and solve for x.

Equation: $\dfrac{145}{x} = \dfrac{145}{x + 40} + \dfrac{1}{5}$

$$145(5)(x + 40) = 145(5)x + x(x + 40)$$

$$725x + 29{,}000 = 725x + x^2 + 40x$$

$$0 = x^2 + 40x - 29{,}000$$

Using the positive value for x found by the Quadratic Formula, we have $x \approx 151.5$ mph and $x + 40 = 191.5$ mph. The airspeed required to obtain the decrease in travel time is 191.5 miles per hour.

92. *Model:* $(\text{Rate}) \cdot (\text{time}) = (\text{distance})$

Labels: Distance $= 1080$

Original time $= t$

Original rate $= \dfrac{1080}{t}$

Return time $= t + 2.5$

Return rate $= \dfrac{1080}{t} - 6$

Equation: $\left(\dfrac{1080}{t} - 6\right)(t + 2.5) = 1080$

$$1080 + \frac{2700}{t} - 6t - 15 = 1080$$
$$\frac{2700}{t} - 6t - 15 = 0$$
$$270 - 0 - 6t^2 - 15t = 0$$
$$2t^2 + 5t - 900 = 0$$
$$(2t + 45)(t - 20) = 0$$
$$2t + 45 = 0 \Rightarrow t = -22.5$$
$$t - 20 = 0 \Rightarrow t = 20$$

The average speed was $\frac{1080}{20} = 54$ miles per hour.

93.
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$2694.58 = 2500\left(1 + \frac{r}{12}\right)^{(12)(5)}$$
$$\frac{2694.58}{2500} = \left(1 + \frac{r}{12}\right)^{60}$$
$$1.077832 = \left(1 + \frac{r}{12}\right)^{60}$$
$$(1.077832)^{1/60} = 1 + \frac{r}{12}$$
$$\left[(1.0.77832)^{1/60} - 1\right](12) = r$$
$$r \approx 0.015 = 1.5\%$$

94.
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$7734.27 = 6000\left(1 + \frac{r}{4}\right)^{(4)(5)}$$
$$\frac{7734.27}{6000} = \left(1 + \frac{r}{4}\right)^{20}$$
$$1.289045 = \left(1 + \frac{r}{4}\right)^{20}$$
$$(1.289045)^{1/20} = 1 + \frac{r}{4}$$
$$\left[(1.289045)^{1/20}\right](4) = r$$
$$r \approx 0.0511 = 5.1\%$$

95. When $C = 2.5$ we have:
$$2.5 = \sqrt{0.2x + 1}$$
$$6.25 = 0.2x + 1$$
$$5.25 = 0.2x$$
$$x = 26.25 = 26{,}250 \text{ passengers}$$

96. $N = \sqrt{4241.855 + 1404.727t},\ 13 \le t \le 18$

(a)
$$160 = \sqrt{4241.855 + 1404.727t}$$
$$(160)^2 = \left(\sqrt{4241.855 + 1404.727t}\right)^2$$
$$25{,}600 = 4241.855 + 1404.727t$$
$$21{,}358.145 = 1404.727t$$
$$15.20 \approx t$$

The number of students passing reached 160,000 in 2015.

(b)
$$170 = \sqrt{4241.855 + 1404.727t}$$
$$(170)^2 = \left(\sqrt{4241.855 + 1404.727t}\right)^2$$
$$28{,}900 = 4241.855 + 1404.727t$$
$$24{,}658.145 = 1404.727t$$
$$17.55 \approx t$$

The number of students passing reached 170,000 in 2017.

97. $T = 75.82 - 2.11x + 43.51\sqrt{x},\ 5 \le x \le 40$

(a) $212 = 75.82 - 2.11x + 43.51\sqrt{x}$

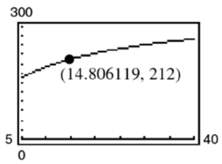
$0 = -2.11x + 43.51\sqrt{x} - 136.18$

By the Quadratic Formula, we have $\sqrt{x} \approx 16.77928 \Rightarrow x \approx 281.333$

$\sqrt{x} \approx 3.84787 \Rightarrow x \approx 14.806.$

Since x is restricted to $5 \le x \le 40$, let $x = 14.806$ pounds per square inch.

(b)



98. $P = \dfrac{218.22 + 0.9052t^2}{1 + 0.0031t^2},\ 10 \le t \le 18$

(a)
$$240 = \frac{218.22 + 0.9052t^2}{1 + 0.0031t^2}$$
$$240 + 0.744t^2 = 218.22 + 0.9052t^2$$
$$21.78 = 0.1612t^2$$
$$\frac{21.78}{0.1612} = t^2$$
$$11.62 \approx t$$

The total voting age population reached 240 million in 2011.

(b)
$$250 = \frac{218.22 + 0.9052t^2}{1 + 0.0031t^2}$$
$$250 + 0.775t^2 = 218.22 + 0.9052t^2$$
$$31.78 = 0.1302t^2$$
$$\frac{31.78}{0.1302} = t^2$$
$$15.62 \approx t$$

The total voting age population reached 250 million in 2015.

99.
$$37.55 = 40 - \sqrt{0.01x + 1}$$
$$\sqrt{0.01x + 1} = 2.45$$
$$0.01x + 1 = 6.0025$$
$$0.01x = 5.0025$$
$$x = 500.25$$

Rounding x to the nearest whole unit yields $x \approx 500$ units.

100. *Verbal Model:* Total cost = Cost underwater · Distance underwater + Cost overland · Distance overland

Labels: Total cost: \$1,098,662.40

Cost overland: \$24 per foot

Distance overland in feet: $5280(8 - x)$

Cost underwater: \$30 per foot

Distance underwater in feet: $5280\sqrt{x^2 + (3/4)^2} = 5280\sqrt{\frac{16x^2 + 9}{16}} = 1320\sqrt{16x^2 + 9}$

Equation:

$$1{,}098{,}662.40 = 30\left(1320\sqrt{16x^2 + 9}\right) + 24\left[5280(8 - x)\right]$$

$$1{,}098{,}662.40 = 39{,}600\sqrt{16x^2 + 9} + 126{,}720(8 - x)$$

$$1{,}098{,}662.40 = 7920\left[5\sqrt{16x^2 + 9} + 16(8 - x)\right]$$

$$138.72 = 5\sqrt{16x^2 + 9} + 16(8 - x)$$

$$138.72 = 5\sqrt{16x^2 + 9} + 128 - 16x$$

$$16x + 10.72 = 5\sqrt{16x^2 + 9}$$

$$(16x + 10.72)^2 = \left(5\sqrt{16x^2 + 9}\right)^2$$

$$256x^2 + 343.04x + 114.9184 = 25(16x^2 + 9)$$

$$256x^2 + 343.04x + 114.9184 = 400x^2 + 225$$

$$0 = 144x^2 - 343.04x + 110.0816$$

By the Quadratic Formula, $x \approx 2$ or $x \approx 0.382$.

So, the length of x could either be 0.382 mile or 2 miles.

101.

$$\frac{1}{t} + \frac{1}{t + 3} = \frac{1}{y}$$

$$\frac{1}{t} + \frac{1}{t + 3} = \frac{1}{2}$$

$$2t(t + 3)\frac{1}{t} + 2t(t + 3)\frac{1}{t + 3} = 2t(t + 3)\frac{1}{2}$$

$$2(t + 3) + 2t = t(t + 3)$$

$$2t + 6 + 2t = t^2 + 3t$$

$$0 = t^2 - t - 6$$

$$0 = (t - 3)(t + 2)$$

$$t - 3 = 0 \Rightarrow t = 3$$

$$t + 2 = 0 \Rightarrow t = -2$$

Since t represents time, $t = 3$ is the only solution. It takes 3 hours for you working alone to tile the floor.

102.

$$\frac{1}{t} + \frac{1}{t+2} = \frac{1}{y}$$

$$\frac{1}{t} + \frac{1}{t+2} = \frac{1}{3}$$

$$3t(t+2)\left(\frac{1}{t}\right) + 3t(t+2)\left(\frac{1}{t+2}\right) = 3t(t+2)\left(\frac{1}{3}\right)$$

$$3(t+2) + 3t = t(t+2)$$

$$3t + 6 + 3t = t^2 + 2t$$

$$0 = t^2 - 4t - 6$$

$$t = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{40}}{2}$$

$$= \frac{4 \pm 2\sqrt{10}}{2}$$

$$= \frac{2(2 \pm \sqrt{10})}{2}$$

$$= 2 \pm \sqrt{10}$$

$$t \approx 5.2 \text{ or } t \approx -1.2$$

Since t represents time, $t \approx 5.2$ is the only solution. It takes approximately 5.2 hours for you working alone to paint the fence.

103.

$$d = \sqrt{\frac{2U}{k}}$$

$$d^2 = \frac{2U}{k}$$

$$d^2k = 2U$$

$$\frac{kd^2}{2} = U$$

104.

$$C = 2\pi\sqrt{\frac{a^2 + b^2}{2}}$$

$$C^2 = \left(2\pi\sqrt{\frac{a^2 + b^2}{2}}\right)^2$$

$$\frac{C^2}{4\pi^2} = \frac{a^2 + b^2}{2}$$

$$\frac{2C^2}{4\pi^2} = a^2 + b^2$$

$$\frac{C^2}{2\pi^2} - b^2 = a^2$$

$$a = \pm\sqrt{\frac{C^2}{2\pi^2} - b^2}$$

105. False. See Example 7 on page 125.

106.

$$\sqrt{x+10} - \sqrt{x-10} = 0$$

$$\sqrt{x+10} = \sqrt{x-10}$$

$$x + 10 = x - 10$$

$$10 \neq -10$$

True. There is no value to satisfy this equation.

107. The quadratic equation was not written in general form before the values of a, b, and c were substituted in the Quadratic Formula. As a result, the substitutions in the Quadratic Formula are incorrect.

108. (a) The formula for volume of the glass cube is V = Length × Width × Height. The volume of water in the cube is the length × width × height of the water. So, the volume is $x \cdot x \cdot (x - 3) = x^2(x - 3)$.

(b) Given the equation $x^2(x - 3) = 320$. The dimensions of the glass cube can be found by solving for x. Then, substitute that value into the expression x^3 to find the volume of the cube.

109. The distance between $(3, -5)$ and $(x, 7)$ is 13.

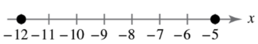
$$\sqrt{(x-3)^2 + (-5-7)^2} = 13$$
$$(x-3)^2 + (-12)^2 = 13^2$$
$$x^2 - 6x + 9 + 144 = 169$$
$$x^2 - 6x - 16 = 0$$
$$(x-8)(x+2) = 0$$
$$x - 8 = 0 \Rightarrow x = 8$$
$$x + 2 = 0 \Rightarrow x = -2$$

Both $(8, 7)$ and $(-2, 7)$ are a distance of 13 from $(3, -5)$.

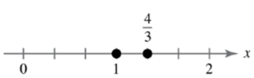
110. The distance between $(10, y)$ and $(4, -3)$ is 10.

$$\sqrt{(10-4)^2 + (y-(-3))^2} = 10$$
$$(6)^2 + (y+3)^2 = 10^2$$
$$36 + (y+3)^2 = 100$$
$$(y+3)^2 = 64$$
$$y + 3 = \pm 8$$
$$y = -3 \pm 8 = -11, 5$$

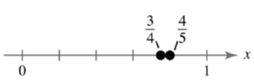
Both $(10, -11)$ and $(10, 5)$ are a distance of 10 from $(4, -3)$.

111. 

$-5 > -12$

112. 

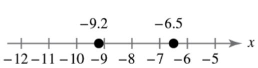
$1 < \frac{4}{3}$

113. 

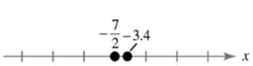
$\frac{3}{4} < \frac{4}{5}$

114. 

$\frac{-9}{8} < \frac{-3}{8}$

115. 

$-6.5 > -9.2$

116. 

$\frac{-7}{2} < -3.4$

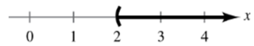
117. The inequality $x \le 3$ denotes the set of all real numbers less than or equal to 3.

118. The inequality $x > 0$ denotes the set of all real numbers greater than 0.

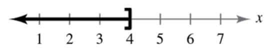
119. The inequality $-5 < x < 5$ denotes the set of all real numbers greater than –5 and less than 5.

120. The inequality $0 < x \le 12$ denotes the set of all positive real numbers less than or equal to 12.

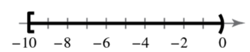
121. The interval $(2, \infty)$ denotes the set of all real numbers greater than 2; $x > 2$.



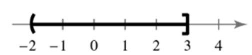
122. The interval $(-\infty, 4]$ denotes the set of all real numbers less than or equal to 4; $x \le 4$.



123. The interval $[-10, 0)$ denotes the set of all real numbers greater than or equal to -10 and less than 0; $-10 \le x < 0$.



124. The interval $(-2, 3]$ denotes the set of all real numbers greater than –2 and less than or equal to 3; $-2 < x \le 3$.



125. $5x - 7 = 3x + 9$
$$2x = 16$$
$$x = 8$$

126. $7x - 3 = 2x + 7$
$$5x = 10$$
$$x = 2$$

127. $1 - \frac{3}{2}x = x - 4$
$$5 = x + \frac{3}{2}x$$
$$5 = \frac{5}{2}x$$
$$x = 2$$

128. $2 - \frac{5}{3}x = x - 6$
$$8 = x + \frac{5}{3}x$$
$$8 = \frac{8}{3}x$$
$$x = 3$$

Section 1.7 Linear Inequalities in One Variable

1. solution set

2. graph

3. double

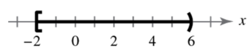
4. union

5. The inequalities $x - 4 < 5$ and $x > 9$ are not equivalent. The first simplifies to $x - 4 < 5 \Rightarrow x < 9$, which is not equivalent to the second, $x > 9$.

6. The Transitive Property of Inequalities is as follows:

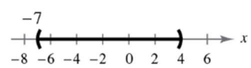
$a < b$ and $b < c \Rightarrow a < c$.

7. Interval: $[-2, 6)$



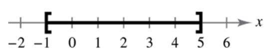
Inequality: $-2 \le x < 6$; The interval is bounded.

8. Interval: $(-7, 4)$



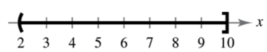
Inequality: $-7 < x < 4$; The interval is bounded.

9. Interval: $[-1, 5]$



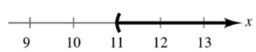
Inequality: $-1 \le x \le 5$; The interval is bounded.

10. Interval: $(2, 10]$



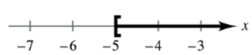
Inequality: $2 < x \le 10$; The interval is bounded.

11. Interval: $(11, \infty)$



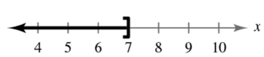
Inequality: $x > 11$; The interval is unbounded.

12. Interval: $[-5, \infty)$



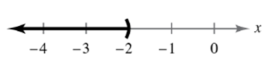
Inequality: $-5 \le x < \infty$ or $x \ge -5$; The interval is unbounded.

13. Interval: $(-\infty, 7]$



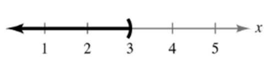
Inequality: $-\infty < x \le 7$ or $x \le 7$; The interval is unbounded.

14. Interval: $(-\infty, -2)$

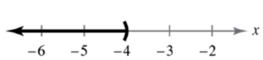


Inequality: $x < -2$; The interval is unbounded.

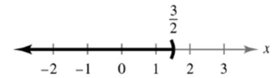
15.
$$4x < 12$$
$$\tfrac{1}{4}(4x) < \tfrac{1}{4}(12)$$
$$x < 3$$



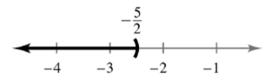
16.
$$10x < -40$$
$$x < -4$$



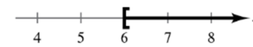
17.
$$-2x > -3$$
$$-\tfrac{1}{2}(-2x) < \left(-\tfrac{1}{2}\right)(-3)$$
$$x < \tfrac{3}{2}$$



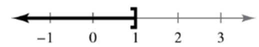
18.
$$-6x > 15$$
$$x < -\tfrac{15}{6} \text{ or } x < -\tfrac{5}{2}$$



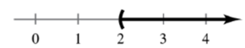
19.
$$2x - 5 \ge 7$$
$$2x \ge 12$$
$$x \ge 6$$



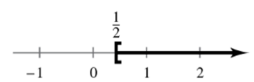
20.
$$5x + 7 \le 12$$
$$5x \le 5$$
$$x \le 1$$



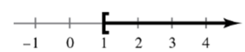
21.
$$2x + 7 < 3 + 4x$$
$$-2x < -4$$
$$x > 2$$



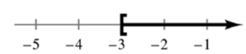
22.
$$3x + 1 \ge 2 + x$$
$$2x \ge 1$$
$$x \ge \tfrac{1}{2}$$



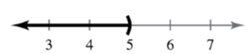
23.
$$3x - 4 \ge 4 - 5x$$
$$8x \ge 8$$
$$x \ge 1$$



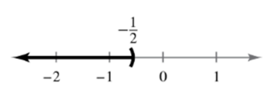
24.
$$6x - 4 \le 2 + 8x$$
$$-2x \le 6$$
$$x \ge -3$$



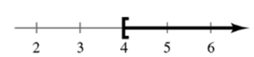
25.
$$4 - 2x < 3(3 - x)$$
$$4 - 2x < 9 - 3x$$
$$x < 5$$



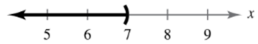
26.
$$4(x + 1) < 2x + 3$$
$$4x + 4 < 2x + 3$$
$$2x < -1$$
$$x < -\tfrac{1}{2}$$



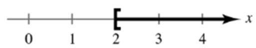
27.
$$\tfrac{3}{4}x - 6 \le x - 7$$
$$-\tfrac{1}{4}x \le -1$$
$$x \ge 4$$



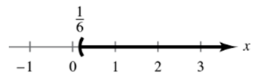
28. $3 + \frac{2}{7}x > x - 2$
$21 + 2x > 7x - 14$
$-5x > -35$
$x < 7$



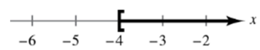
29. $\frac{1}{2}(8x + 1) \geq 3x + \frac{5}{2}$
$4x + \frac{1}{2} \geq 3x + \frac{5}{2}$
$x \geq 2$



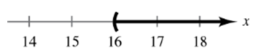
30. $9x - 1 < \frac{3}{4}(16x - 2)$
$36x - 4 < 48x - 6$
$-12x < -2$
$x > \frac{1}{6}$



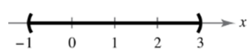
31. $3.6x + 11 \geq -3.4$
$3.6x \geq 14.4$
$x \geq -4$



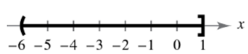
32. $15.6 - 1.3x < -5.2$
$-1.3x < -20.8$
$x > 16$



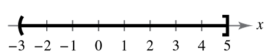
33. $1 < 2x + 3 < 9$
$-2 < 2x < 6$
$-1 < x < 3$



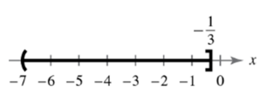
34. $-9 \leq -2x - 7 < 5$
$-2 \leq -2x < 12$
$1 \geq x > -6$
$-6 < x \leq 1$



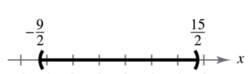
35. $-1 \leq -(x - 4) < 7$
$1 \geq x - 4 > -7$
$5 \geq x > -3$
$-3 < x \leq 5$



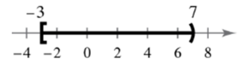
36. $0 < 3(x + 7) \leq 20$
$0 < x + 7 \leq \frac{20}{3}$
$-7 < x \leq -\frac{1}{3}$



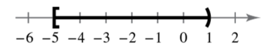
37. $-4 < \frac{2x - 3}{3} < 4$
$-12 < 2x - 3 < 12$
$-9 < 2x < 15$
$-\frac{9}{2} < x < \frac{15}{2}$



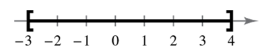
38. $0 \leq \frac{x + 3}{2} < 5$
$0 \leq x + 3 < 10$
$-3 \leq x < 7$



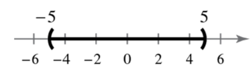
39. $-1 < \frac{-x - 2}{3} \leq 1$
$-3 < -x - 2 \leq 3$
$-1 < -x \leq 5$
$1 > x \geq -5$
$-5 \leq x < 1$



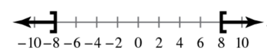
40. $-1 \leq \frac{-3x + 5}{7} \leq 2$
$-7 \leq -3x + 5 \leq 14$
$-12 \leq -3x \leq 9$
$4 \geq x \geq -3$
$-3 \leq x \leq 4$



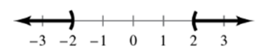
41. $|x| < 5$
$-5 < x < 5$



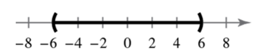
42. $|x| \geq 8$
$x \geq 8$ or $x \leq -8$



43. $\left|\frac{x}{2}\right| > 1$
$\frac{x}{2} < -1$ or $\frac{x}{2} > 1$
$x < -2$ $\quad$ $x > 2$



44. $\left|\frac{x}{3}\right| < 2$
$-2 < \frac{x}{3} < 2$
$-6 < x < 6$



45. $|x - 5| < -1$

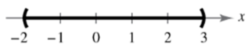
No solution. The absolute value of a number cannot be less than a negative number.

46. $|x - 7| < -5$

No solution. The absolute value of a number cannot be less than a negative number.

47. $|7 - 2x| \geq 9$
$7 - 2x \leq -9$ or $7 - 2x \geq 9$
$-2x \leq -16$ $\quad$ $-2x \geq 2$
$x \geq 8$ $\quad$ $x \leq -1$

48. $|1 - 2x| < 5$



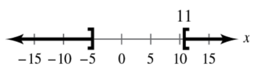
$-5 < 1 - 2x < 5$

$-6 < -2x < 4$

$3 > x > -2$

$-2 < x < 3$

49. $\left|\dfrac{x-3}{2}\right| \geq 4$

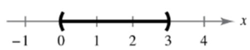


$\dfrac{x-3}{2} \leq -4$ or $\dfrac{x-3}{2} \geq 4$

$x - 3 \leq -8$ $\quad$ $x - 3 \geq 8$

$x \leq -5$ $\quad$ $x \geq 11$

50. $\left|1 - \dfrac{2x}{3}\right| < 1$



$-1 < 1 - \dfrac{2x}{3} < 1$

$-2 < -\dfrac{2x}{3} < 0$

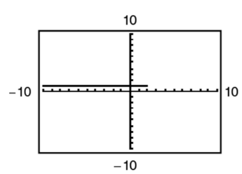
$3 > x > 0$

$0 < x < 3$

51. $8 - 3x \geq 2$

$-3x \geq -6$

$x \leq 2$

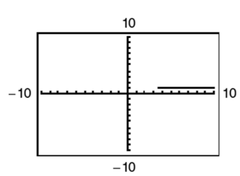


52. $20 < 6x - 1$

$21 < 6x$

$\frac{7}{2} < x$

$x > 3.5$

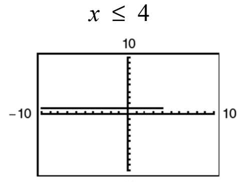


53. $4(x - 3) \leq 8 - x$

$4x - 12 \leq 8 - x$

$5x \leq 20$

$x \leq 4$

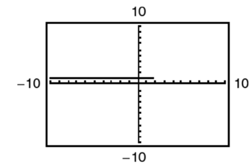


54. $3(x + 1) < x + 7$

$3x + 3 < x + 7$

$2x < 4$

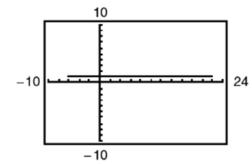
$x < 2$



55. $|x - 8| \leq 14$

$-14 \leq x - 8 \leq 14$

$-6 \leq x \leq 22$

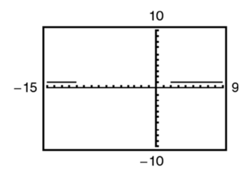


56. $|2x + 9| > 13$

$2x + 9 < -13$ or $2x + 9 > 13$

$2x < -22$ $\quad$ $2x > 4$

$x < -11$ $\quad$ $x > 2$

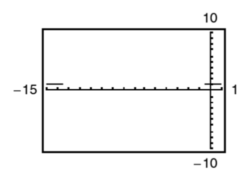


57. $2|x + 7| \geq 13$

$|x + 7| \geq \frac{13}{2}$

$x + 7 \leq -\frac{13}{2}$ or $x + 7 \geq \frac{13}{2}$

$x \leq -\frac{27}{2}$ $\quad$ $x \geq -\frac{1}{2}$

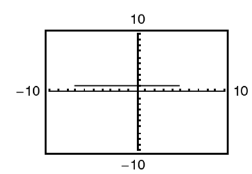


58. $\frac{1}{2}|x + 1| \leq 3$

$|x + 1| \leq 6$

$-6 \leq x + 1 \leq 6$

$-7 \leq x \leq 5$



59. $y = 3x - 1$

(a) $y \geq 2$

$3x - 1 \geq 2$

$3x \geq 3$

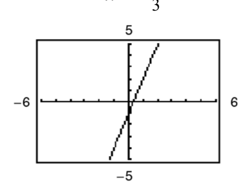
$x \geq 1$

(b) $y \leq 0$

$3x - 1 \leq 0$

$3x \leq 1$

$x \leq \frac{1}{3}$



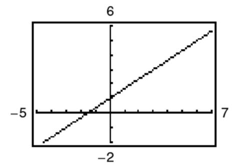
60. $y = \frac{2}{3}x + 1$

(a) $y \le 5$

$\frac{2}{3}x + 1 \le 5$

$\frac{2}{3}x \le 4$

$x \le 6$



(b) $y \ge 0$

$\frac{2}{3}x + 1 \ge 0$

$\frac{2}{3}x \ge -1$

$x \ge -\frac{3}{2}$

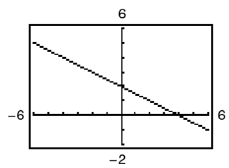
61. $y = -\frac{1}{2}x + 2$

(a) $0 \le y \le 3$

$0 \le -\frac{1}{2}x + 2 \le 3$

$-2 \le -\frac{1}{2}x \le 1$

$4 \ge x \ge -2$



(b) $y \ge 0$

$-\frac{1}{2}x + 2 \ge 0$

$-\frac{1}{2}x \ge -2$

$x \le 4$

62. $y = -3x + 8$

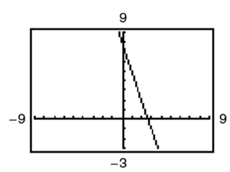
(a) $-1 \le y \le 3$

$-1 \le -3x + 8 \le 3$

$-9 \le -3x \le -5$

$3 \ge x \ge \frac{5}{3}$

$\frac{5}{3} \le x \le 3$



(b) $y \le 0$

$-3x + 8 \le 0$

$-3x \le -8$

$x \ge \frac{8}{3}$

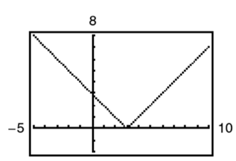
63. $y = |x - 3|$

(a) $y \le 2$

$|x - 3| \le 2$

$-2 \le x - 3 \le 2$

$1 \le x \le 5$



(b) $y \ge 4$

$|x - 3| \ge 4$

$x - 3 \le -4$ or $x - 3 \ge 4$

$x \le -1$ or $x \ge 7$

64. $y = \left|\frac{1}{2}x + 1\right|$

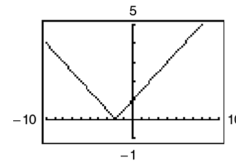
(a) $y \le 4$

$\left|\frac{1}{2}x + 1\right| \le 4$

$-4 \le \frac{1}{2}x + 1 \le 4$

$-5 \le \frac{1}{2}x \le 3$

$-10 \le x \le 6$



(b) $y \ge 1$

$\left|\frac{1}{2}x + 1\right| \ge 1$

$\frac{1}{2}x + 1 \le -1$ or $\frac{1}{2}x + 1 \ge 1$

$\frac{1}{2}x \le -2$ $\quad$ $\frac{1}{2}x \ge 0$

$x \le -4$ $\quad$ $x \ge 0$

65. The midpoint of the interval $[-3, 3]$ is 0. The interval represents all real numbers x no more than 3 units from 0.

$|x - 0| \le 3$

$|x| \le 3$

66. The graph shows all real numbers more than 3 units from 0.

$|x - 0| > 3$

$|x| > 3$

67. The graph shows all real numbers at least 3 units from 7.

$|x - 7| \ge 3$

68. The graph shows all real numbers no more than 4 units from -1.

$|x + 1| \le 4$

69. All real numbers less than 3 units from 7

$|x - 7| \ge 3$

70. All real numbers at least 5 units from 8

$|x - 8| \ge 5$

71. All real numbers less than 4 units from -3

$|x - (-3)| < 4$

$|x + 3| < 4$

72. All real numbers no more than 7 units from -6

$|x + 6| \le 7$

73. $\$7.25 \le P \le \$7.75$

74. $180 < w < 185.5$

75. $r \le 0.08$

76. $I \ge \$239{,}000{,}000$

77. $r = 220 - A = 220 - 20 = 200$ beats per minute

$0.50(200) \le r \le 0.85(200)$

$100 \le r \le 170$

The target heart rate is at least 100 beats per minute and at most 170 beats per minute.

78. $r = 220 - A = 220 - 40 = 180$ beats per minute

$0.50(180) \le r \le 0.85(180)$

$90 \le r \le 153$

The target heartrate is at least 90 beats per minute and at most 153 beats per minute.

79. $9.00 + 0.75x > 13.50$

$0.75x > 4.50$

$x > 6$

You must produce at least 6 units each hour in order to yield a greater hourly wage at the second job.

80. $10.00 + 1.25x > 13.75$

$1.25x > 3.75$

$x > 3$

You must produce more than 3 units each hour in order to yield a greater hourly wage at the second job.

81. $1000(1 + r(10)) > 2000.00$

$1 + 10r > 2$

$10r > 1$

$r > 0.1$

The rate must be greater than 10%.

82. $750 < 500(1 + r(5))$

$1.5 < 1 + 5r$

$0.5 < 5r$

$0.1 < r$

The rate must be more than 10%.

83. $R > C$

$115.95x > 95x + 750$

$20.95x > 750$

$x \ge 35.7995$

$x \ge 36$ units

84. $24.55x > 15.4x + 150{,}000$

$9.15 > 150{,}000$

$x > 16{,}393.44262$

Because the number of units x must be an integer, the product will return a profit when at least 16,394 units are sold.

85. Let x = number of dozen doughnuts sold per day.

Revenue: $R = 7.95x$

Cost: $C = 1.45x + 165$

$P = R - C$

$= 7.95x - (1.45x + 165)$

$= 6.50x - 165$

$400 \le P \le 1200$

$400 \le 6.50x - 165 \le 1200$

$565 \le 6.50x \le 1365$

$86.9 \le x \le 210$

The daily sales vary between 87 and 210 dozen doughnuts per day.

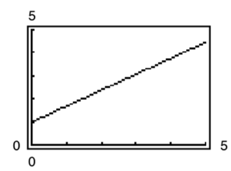
86. The goal is to lose $164 - 128 = 36$ pounds. At $1\frac{1}{2}$ pounds per week, it will take 24 weeks.

$36 \div 1\frac{1}{2} = 36 \times \frac{2}{3}$

$= 12 \times 2$

$= 24$

87. (a)



(b) From the graph you see that $y \ge 3$ when $x \ge 2.9$.

(c) Algebraically:

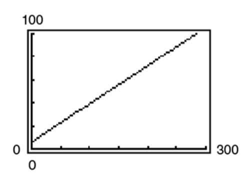
$3 \le 0.692x + 0.988$

$2.012 \le 0.692x$

$2.91 \le x$

$x \ge 2.91$

88. (a)



(b) One estimate is $x \le 224$ pounds.

(c) $0.33x + 6.20 \le 80$

$0.33x \le 73.8$

$x \le 223.636$

89. $W = 0.693t + 32.23,\ 10 \le t \le 18$

(a) $40 \le 0.693t + 32.23 \le 42$

$7.77 \le 0.693t \le 9.77$

$11.21 \le t \le 14.10$

Between 2011 and 2014, the mean hourly wage was at least \$40, but no more than \$42.

(b) $0.693t + 32.23 > 44$

$0.693t > 11.77$

$t > 16.98$

The mean hourly wage exceeded \$44 in 2016.

90. $M = 3.125t + 161.93,\ 10 \le t \le 18$

(a) $200 < 3.125t + 161.93 \le 210$

$38.07 < 3.125t \le 48.07$

$12.18 < t \le 15.38$

Between 2012 and 2015, the annual milk production was greater than 200 billion pounds, but no more than 210 billion pounds.

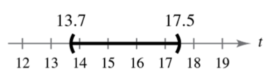
(b) $3.125t + 161.93 > 212$

$3.125t > 50.07$

$t > 16.02$

Milk production exceeded 212 billion pounds in 2016.

91. $\left|\dfrac{t - 15.6}{1.9}\right| < 1$



$-1 < \dfrac{t - 15.6}{1.9} < 1$

$-1.9 < t - 15.6 < 1.9$

$13.7 < t < 17.5$

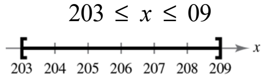
Two-thirds of the workers could perform the task in the time interval between 13.7 minutes and 17.5 minutes.

92. (a) $|x - 206| \le 3$

$-3 \le x - 206 \le 3$

$203 \le x \le 09$

(b)



93. $1 \text{ oz} = \dfrac{1}{16}$ lb, so $\dfrac{1}{2}$ oz $= \dfrac{1}{32}$ lb.

Because $8.99 \cdot \dfrac{1}{32} = 0.2809375$, you may be undercharged or overcharged by \$0.28.

94. $24.2 - 0.25 \le s \le 24.2 + 0.25$

$23.95 \le s \le 24.45$

The interval containing the possible side lengths s in centimeters of the square is $[23.95, 24.45]$, so the interval containing the possible areas in square centimeters is $\left[23.95^2, 24.45^2\right]$, or $[573.6025, 597.8025]$.

95. True. This is the Addition of a Constant Property of Inequalities.

96. False. If c is negative, then $ac \ge bc$.

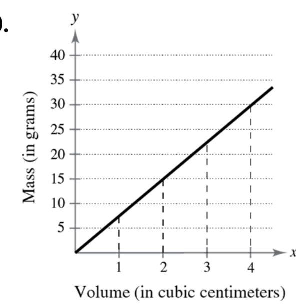
97. False. If $-10 \le x \le 8$, then $10 \ge -x$ and $-x \ge -8$.

98. True.

Because $|2x - 5| \ge 0$, the only solution of $|2x - 5| \le 0$ is $2x - 5 = 0$, or $x = \frac{5}{2}$.

99. Answer not unique. Sample answer: $x < x + 1$

100.



(a) When the volume is 2 cubic centimeters, the mass is approximately 15 grams.

(b) When the volume is greater than or equal to 0 cubic centimeters, and less than 4 cubic centimeters, $0 \le x < 4$ the mass is greater than or equal to 0 grams and less than 30 grams, $0 \le y < 30$.

101. $|3x - 4|$ is always greater than or equal to 0, so the inequality is true for all real numbers.

102. (a) $500(1+r)^2 = 500(r+1)^2$

$= 500(r^2 + 2r + 1)$

$= 500r^2 + 1000r + 500$

(b)

r	$2\frac{1}{2}\%$	3%	4%	$4\frac{1}{2}\%$	5%
$500(1+r)^2$	\$525.31	\$530.45	\$540.80	\$546.01	\$551.25

(a) As r increases, the amount increases.

103.
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x = 3, -2$$

104.
$$x^2 - x - 20 = 0$$
$$(x-5)(x+4) = 0$$
$$x = 5, -4$$

105.
$$4x^2 - 5x = 6$$
$$4x^2 - 5x - 6 = 0$$
$$(4x+3)(x-2) = 0$$
$$x = -\tfrac{3}{4}, 2$$

106.
$$2x^2 + 3x = 5$$
$$2x^2 + 3x - 5 = 0$$
$$(2x+5)(x-1) = 0$$
$$x = -\tfrac{5}{2}, 1$$

107.
$$2x^3 - 3x^2 = 32x - 48$$
$$2x^3 - 3x^2 - 32x + 48 = 0$$
$$x^2(2x-3) - 16(2x-3) = 0$$
$$(x^2 - 16)(2x-3) = 0$$
$$(x-4)(x+4)(2x-3) = 0$$
$$x = \pm 4, \tfrac{3}{2}$$

108.
$$3x^3 - x^2 = 12x - 4$$
$$3x^3 - x^2 - 12x + 4 = 0$$
$$3x^3 - 12x - x^2 + 4 = 0$$
$$3x(x^2 - 4) - (x^2 - 4) = 0$$
$$(x^2 - 4)(3x - 1) = 0$$
$$x = \pm 2, \tfrac{1}{3}$$

109.
$$\frac{2x-7}{x-5} = 3$$
$$2x - 7 = 3x - 15$$
$$8 = x$$

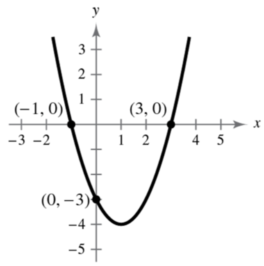
110.
$$\frac{1}{x-3} = \frac{3}{2x+1}$$
$$2x + 1 = 3(x-3)$$
$$2x + 1 = 3x - 9$$
$$10 = x$$

111. $y = x^2 - 2x - 3 = (x - 3)(x + 1)$

$y = (-x)^2 - 2(-x) - 3 \Rightarrow y = x^2 + 2x - 3 \Rightarrow$ No y-axis symmetry

$-y = x^2 - 2x - 3 \Rightarrow y = -x^2 + 2x + 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^2 - 2(-x) - 3 \Rightarrow -y = x^2 + 2x - 3 = y = -x^2 - 2x + 3 \Rightarrow$ No origin symmetry



x-intercepts: $(-1, 0)(3, 0)$

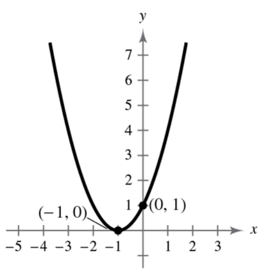
y-intercept: $(0, -3)$

112. $y = x^2 + 2x + 1 = (x + 1)^2$

$y = (-x)^2 + 2(-x) + 1 \Rightarrow y = x^2 - 2x + 1 \Rightarrow$ No y-axis symmetry

$-y = x^2 + 2x + 1 \Rightarrow y = -x^2 - 2x - 1 \Rightarrow$ No x-axis symmetry

$-y = (-x)^2 + 2(-x) + 1 \Rightarrow -y = x^2 - 2x + 1 \Rightarrow y = -x^2 + 2x - 1 \Rightarrow$ No origin symmetry



x-intercept: $(-1, 0)$

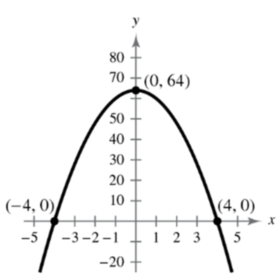
y-intercept: $(0, 1)$

113. $y = 64 - 4x^2$

$y = 64 - 4(-x)^2 \Rightarrow y = 64 - 4x^2 \Rightarrow$ y-axis symmetry

$-y = 64 - 4x^2 \Rightarrow y = -64 + 4x^2 \Rightarrow$ No x-axis symmetry

$-y = 64 - 4(-x)^2 \Rightarrow -y = 64 - 4x^2 \Rightarrow y = -64 + 4x^2 \Rightarrow$ No origin symmetry



x-intercepts: $(\pm 4, 0)$

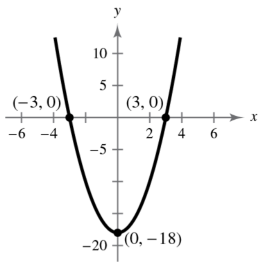
y-intercept: $(0, 64)$

114. $y = 2x^2 - 18$

$y = 2(-x)^2 - 18 \Rightarrow y = 2x^2 - 18 \Rightarrow$ y-axis symmetry

$-y = 2x^2 - 18 \Rightarrow y = -2x^2 + 18 \Rightarrow$ No x-axis symmetry

$-y = 2(-x)^2 - 18 \Rightarrow -y = 2x^2 - 18 \Rightarrow y = -2x^2 + 18 \Rightarrow$ No origin symmetry



x-intercepts: $(\pm 3,\ 0)$

y-intercept: $(0,\ -18)$

Section 1.8 Other Types of Inequalities

1. positive; negative

2. zeros; undefined values

3. The key numbers of the inequality are -2 and 5.

4. No. $4(4 - 4) = 0$, which is not less than 0.

5. $x^2 - 3 < 0$

(a) $x = 3$

$(3)^2 - 3 \stackrel{?}{<} 0$

$6 \not< 0$

No, $x = 3$ *is not* a solution.

(b) $x = 0$

$(0)^2 - 3 \stackrel{?}{<} 0$

$-3 < 0$

Yes, $x = 0$ *is* a solution.

(c) $x = \frac{3}{2}$

$\left(\frac{3}{2}\right)^2 - 3 \stackrel{?}{<} 0$

$-\frac{3}{4} < 0$

Yes, $x = \frac{3}{2}$ *is* a solution.

(d) $x = -5$

$(-5)^2 - 3 \stackrel{?}{<} 0$

$22 \not< 0$

No, $x = -5$ *is not* a solution

6. $x^2 - 2x - 8 \ge 0$

(a) $x = 5$

$(5)^2 - 2(5) - 8 \stackrel{?}{\ge} 0$

$7 \ge 0$

Yes, $x = 5$ *is* a solution.

(b) $x = 0$

$(0)^2 - 2(0) - 8 \stackrel{?}{\ge} 0$

$-8 \not\ge 0$

No, $x = 0$ *is not* a solution.

(c) $x = -4$

$(-4)^2 - 2(-4) - 8 \stackrel{?}{\ge} 0$

$16 + 8 - 8 \stackrel{?}{\ge} 0$

$16 \ge 0$

Yes, $x = -4$ *is* a solution.

(d) $x = 1$

$(1)^2 - 2(1) - 8 \stackrel{?}{\ge} 0$

$1 - 2 - 8 \stackrel{?}{\ge} 0$

$-9 \not\ge 0$

No, $x = 1$ *is not* a solution

7. $\dfrac{x+2}{x-4} \ge 3$

(a) $x = 5$

$\dfrac{5+2}{5-4} \overset{?}{\ge} 3$

$7 \ge 3$

Yes, $x = 5$ *is* a solution.

(b) $x = 4$

$\dfrac{4+2}{4-4} \overset{?}{\ge} 3$

$\dfrac{6}{0}$ is undefined.

No, $x = 4$ *is not* a solution.

(c) $x = -\dfrac{9}{2}$

$\dfrac{-\frac{9}{2}+2}{-\frac{9}{2}-4} \overset{?}{\ge} 3$

$\dfrac{5}{17} \not\ge 3$

No, $x = -\dfrac{9}{2}$ *is not* a solution.

(d) $x = \dfrac{9}{2}$

$\dfrac{\frac{9}{2}+2}{\frac{9}{2}-4} \overset{?}{\ge} 3$

$13 \ge 3$

Yes, $x = \dfrac{9}{2}$ *is* a solution.

8. $\dfrac{3x^2}{x^2+4} < 1$

(a) $x = -2$

$\dfrac{3(-2)^2}{(-2)^2+4} \overset{?}{<} 1$

$\dfrac{12}{8} \not< 1$

No, $x = -2$ *is not* a solution.

(b) $x = -1$

$\dfrac{3(-1)^2}{(-1)^2+4} \overset{?}{<} 1$

$\dfrac{3}{5} < 1$

Yes, $x = -1$ *is* a solution.

(c) $x = 0$

$\dfrac{3(0)^2}{(0)^2+4} \overset{?}{<} 1$

$0 < 1$

Yes, $x = 0$ *is* a solution.

(d) $x = 3$

$\dfrac{3(3)^2}{(3)^2+4} \overset{?}{<} 1$

$\dfrac{27}{13} \not< 1$

No, $x = 3$ *is not* a solution.

9. $x^2 - 3x - 18 = (x+3)(x-6)$

$x + 3 = 0 \Rightarrow x = -3$

$x - 6 = 0 \Rightarrow x = 6$

The key numbers are -3 and 6.

10. $9x^3 - 25x^2 = 0$

$x^2(9x - 25) = 0$

$x^2 = 0 \Rightarrow x = 0$

$9x - 25 = 0 \Rightarrow x = \frac{25}{9}$

The key numbers are 0 and $\frac{25}{9}$.

11. $\dfrac{1}{x-5} + 1 = \dfrac{1 + 1(x-5)}{x-5}$

$= \dfrac{x-4}{x-5}$

$x - 4 = 0 \Rightarrow x = 4$

$x - 5 = 0 \Rightarrow x = 5$

The key numbers are 4 and 5.

12. $\dfrac{x}{x+2} - \dfrac{2}{x-1} = \dfrac{x(x-1) - 2(x+2)}{(x+2)(x-1)}$

$= \dfrac{x^2 - x - 2x - 4}{(x+2)(x-1)}$

$= \dfrac{(x-4)(x+1)}{(x+2)(x-1)}$

$(x-4)(x+1) = 0$

$x - 4 = 0 \Rightarrow x = 4$

$x + 1 = 0 \Rightarrow x = -1$

$(x+2)(x-1) = 0$

$x + 2 = 0 \Rightarrow x = -2$

$x - 1 = 0 \Rightarrow x = 1$

The key numbers are $-2, -1, 1,$ and 4.

13. $2x^2 + 4x < 0$

$2x(x + 2) < 0$

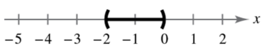
Key numbers: $x = 0, -2$

Test intervals: $(-\infty, -2), (-2, 0), (0, \infty)$

Test: Is $2x(x + 2) < 0$?

Interval	x-Value	Value of $2x(x + 2)$	Conclusion
$(-\infty, -2)$	-3	6	Positive
$(-2, 0)$	-1	-2	Negative
$(0, \infty)$	1	3	Positive

Solution set: $(-2, 0)$



14. $3x^2 - 9x \ge 0$

$3x(x - 3) \ge 0$

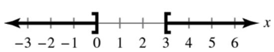
Key numbers: $x = 0, 3$

Test intervals: $(-\infty, 0), (0, 3), (3, \infty)$

Test: Is $3x(x - 3) > 0$?

Interval	x-Value	Value of $3x(x - 3)$	Conclusion
$(-\infty, 0)$	-1	12	Positive
$(0, 3)$	1	-6	Negative
$(3, \infty)$	4	12	Positive

Solution set: $(-\infty, 0] \cup [3, \infty)$



15. $x^2 < 9$

$x^2 - 9 < 0$

$(x + 3)(x - 3) < 0$

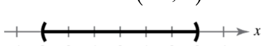
Key numbers: $x = \pm 3$

Test intervals: $(-\infty, -3), (-3, 3), (3, \infty)$

Test: Is $(x + 3)(x - 3) < 0$?

Interval	x-Value	Value of $x^2 - 9$	Conclusion
$(-\infty, -3)$	-4	7	Positive
$(-3, 3)$	0	-9	Negative
$(3, \infty)$	4	7	Positive

Solution set: $(-3, 3)$



16. $x^2 \le 25$

$x^2 - 25 \le 0$

$(x + 5)(x - 5) \le 0$

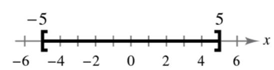
Key numbers: $x = \pm 5$

Test intervals: $(-\infty, -5), (-5, 5), (5, \infty)$

Test: Is $(x + 5)(x - 5) \le 0$?

Interval	x-Value	Value of $x^2 - 25$	Conclusion
$(-\infty, -5)$	-6	11	Positive
$(-5, 5)$	0	-25	Negative
$(5, \infty)$	6	11	Positive

Solution set: $[-5, 5]$



17. $(x + 2)^2 \le 25$

$x^2 + 4x + 4 \le 25$

$x^2 + 4x - 21 \le 0$

$(x + 7)(x - 3) \le 0$

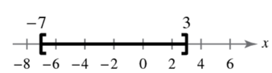
Key numbers: $x = -7, x = 3$

Test intervals: $(-\infty, -7), (-7, 3), (3, \infty)$

Test: Is $(x + 7)(x - 3) \le 0$?

Interval	x-Value	Value of $(x + 7)(x - 3)$	Conclusion
$(-\infty, -7)$	-8	$(-1)(-11) = 11$	Positive
$(-7, 3)$	0	$(7)(-3) = -21$	Negative
$(3, \infty)$	4	$(11)(1) = 11$	Positive

Solution set: $[-7, 3]$



18. $(x-3)^2 \geq 1$

$x^2 - 6x + 8 \geq 0$

$(x-2)(x-4) \geq 0$

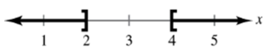
Key numbers: $x = 2, x = 4$

Test intervals: $(-\infty, 2) \Rightarrow (x-2)(x-4) > 0$

$(2, 4) \Rightarrow (x-2)(x-4) < 0$

$(4, \infty) \Rightarrow (x-2)(x-4) > 0$

Solution set: $(-\infty, 2] \cup [4, \infty)$



19. $x^2 + 6x + 1 \geq -7$

$x^2 + 6x + 8 \geq 0$

$(x+2)(x+4) \geq 0$

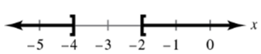
Key numbers: $x = -2, x = -4$

Test Intervals: $(-\infty, -4), (-4, -2), (-2, \infty)$

Test: Is $(x+2)(x+4) > 0$?

Interval	x-Value	Value of $(x+2)(x+4)$	Conclusion
$(-\infty, -4)$	-6	8	Positive
$(-4, -2)$	-3	-1	Negative
$(-2, \infty)$	0	8	Positive

Solution set: $(-\infty, -4] \cup [-2, \infty)$



20. $x^2 - 8x + 2 < 11$

$x^2 - 8x - 9 < 0$

$(x-9)(x+1) < 0$

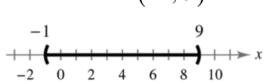
Key numbers: $x = -1, x = 9$

Test intervals: $(-\infty, -1) \Rightarrow (x-9)(x+1) > 0$

$(-1, 9) \Rightarrow (x-9)(x+1) < 0$

$(9, \infty) \Rightarrow (x-9)(x+1) > 0$

Solution set: $(-1, 9)$



21. $x^2 + x < 6$

$x^2 + x - 6 < 0$

$(x+3)(x-2) < 0$

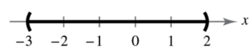
Key numbers: $x = -3, x = 2$

Test intervals: $(-\infty, -3), (-3, 2), (2, \infty)$

Test: Is $(x+3)(x-2) < 0$?

Interval	x-Value	Value of $(x+3)(x-2)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-6) = 6$	Positive
$(-3, 2)$	0	$(3)(-2) = -6$	Negative
$(2, \infty)$	3	$(6)(1) = 6$	Positive

Solution set: $(-3, 2)$



22. $x^2 + 2x > 3$

$x^2 + 2x - 3 > 0$

$(x+3)(x-1) > 0$

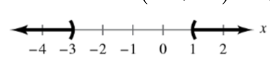
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3) \Rightarrow (x+3)(x-1) > 0$

$(-3, 1) \Rightarrow (x+3)(x-1) < 0$

$(1, \infty) \Rightarrow (x+3)(x-1) > 0$

Solution set: $(-\infty, -3) \cup (1, \infty)$



23. $x^2 < 3 - 2x$

$x^2 + 2x - 3 < 0$

$(x+3)(x-1) < 0$

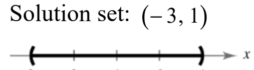
Key numbers: $x = -3, x = 1$

Test intervals: $(-\infty, -3), (-3, 1), (1, \infty)$

Test: Is $(x+3)(x-1) < 0$?

Interval	x-Value	Value of $(x+3)(x-1)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-5) = 5$	Positive
$(-3, 1)$	0	$(3)(-1) = -3$	Negative
$(1, \infty)$	2	$(5)(1) = 5$	Positive

Solution set: $(-3, 1)$



24. $x^2 > 2x + 8$

$x^2 - 2x - 8 > 0$

$(x - 4)(x + 2) > 0$

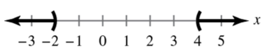
Key numbers: $x = -2, x = 4$

Test intervals: $(-\infty, -2), (-2, 4), (4, \infty)$

Test: Is $(x - 4)(x + 2) > 0$?

Interval	x-Value	Value of $(x - 4)(x + 2)$	Conclusion
$(-\infty, -2)$	-3	$(-7)(-1) = 7$	Positive
$(-2, 4)$	0	$(-4)(2) = -8$	Negative
$(4, \infty)$	5	$(1)(7) = 7$	Positive

Solution set: $(-\infty, -2) \cup (4, \infty)$



25. $3x^2 - 11x > 20$

$3x^2 - 11x - 20 > 0$

$(3x + 4)(x - 5) > 0$

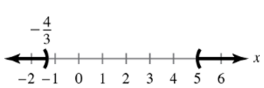
Key numbers: $x = 5, x = -\frac{4}{3}$

Test intervals: $\left(-\infty, -\frac{4}{5}\right), \left(-\frac{4}{3}, 5\right), (5, \infty)$

Test: Is $(3x + 4)(x - 5) > 0$?

Interval	x-Value	Value of $(3x + 4)(x - 5)$	Conclusion
$\left(-\infty, -\frac{4}{3}\right)$	-3	$(-5)(-8) = 40$	Positive
$\left(-\frac{4}{3}, 5\right)$	0	$(4)(-5) = -20$	Negative
$(5, \infty)$	6	$(22)(1) = 22$	Positive

Solution set: $\left(-\infty, -\frac{4}{3}\right) \cup (5, \infty)$



26. $-2x^2 + 6x + 15 \le 0$

$2x^2 - 6x - 15 \ge 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-15)}}{2(2)}$$

$$= \frac{6 \pm \sqrt{156}}{4}$$

$$= \frac{6 \pm 2\sqrt{39}}{4}$$

$$= \frac{3}{2} \pm \frac{\sqrt{39}}{2}$$

Key numbers: $x = \frac{3}{2} - \frac{\sqrt{39}}{2}, x = \frac{3}{2} + \frac{\sqrt{39}}{2}$

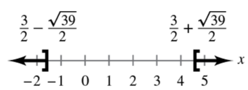
Test intervals:

$\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 < 0$

$\left(\frac{3}{2} - \frac{\sqrt{39}}{2}, \frac{3}{2} + \frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2 + 6x + 15 > 0$

$\left(\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right) \Rightarrow -2x^2 + 6x + 15 < 0$

Solution set: $\left(-\infty, \frac{3}{2} - \frac{\sqrt{39}}{2}\right] \cup \left[\frac{3}{2} + \frac{\sqrt{39}}{2}, \infty\right)$



27.
$$x^3 - 3x^2 - x + 3 > 0$$
$$x^2(x - 3) - (x - 3) > 0$$
$$(x - 3)(x^2 - 1) > 0$$
$$(x - 3)(x + 1)(x - 1) > 0$$

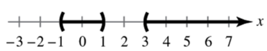
Key numbers: $x = -1, x = 1, x = 3$

Test intervals: $(-\infty, -1), (-1, 1), (1, 3), (3, \infty)$

Test: Is $(x - 3)(x + 1)(x - 1) > 0?$

Interval	x-Value	Value of $(x - 3)(x + 1)(x - 1)$	Conclusion
$(-\infty, -1)$	-2	$(-5)(-1)(-3) = -15$	Negative
$(-1, 1)$	0	$(-3)(1)(-1) = 3$	Positive
$(1, 3)$	2	$(-1)(3)(1) = -3$	Negative
$(3, \infty)$	4	$(1)(5)(3) = 15$	Positive

Solution set: $(-1, 1) \cup (3, \infty)$



28.
$$x^3 + 2x^2 - 4x - 8 \le 0$$
$$x^2(x + 2) - 4(x + 2) \le 0$$
$$(x + 2)(x^2 - 4) \le 0$$
$$(x + 2)^2(x - 2) \le 0$$

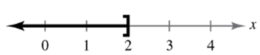
Key numbers: $x = -2, x = 2$

Test intervals: $(-\infty, -2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(-2, 2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(2, \infty) \Rightarrow x^3 + 2x^2 - 4x - 8 > 0$

Solution set: $(-\infty, 2]$



29.
$$-x^3 + 7x^2 + 9x > 63$$
$$x^3 - 7x^2 - 9x < -63$$
$$x^3 - 7x^2 - 9x + 63 < 0$$
$$x^2(x - 7) - 9(x - 7) < 0$$
$$(x - 7)(x^2 - 9) < 0$$
$$(x - 7)(x + 3)(x - 3) < 0$$

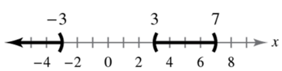
Key numbers: $x = -3, x = 3, x = 7$

Test intervals: $(-\infty, -3), (-3, 3), (3, 7), (7, \infty)$

Test: Is $(x - 7)(x + 3)(x - 3) < 0$?

Interval	x-Value	Value of $(x - 7)(x + 3)(x - 3)$	Conclusion
$(-\infty, -3)$	-4	$(-11)(-1)(-7) = -77$	Negative
$(-3, 3)$	0	$(-7)(3)(-3) = 63$	Positive
$(3, 7)$	4	$(-3)(7)(1) = -21$	Negative
$(7, \infty)$	8	$(1)(11)(5) = 55$	Positive

Solution set: $(-\infty, -3) \cup (3, 7)$



30.
$$2x^3 + 13x^2 - 8x - 46 \ge 6$$
$$2x^3 + 13x^2 - 8x - 52 \ge 0$$
$$x^2(2x + 13) - 4(2x + 13) \ge 0$$
$$(2x + 13)(x^2 - 4) \ge 0$$
$$(2x + 13)(x + 2)(x - 2) \ge 0$$

Key numbers: $x = -\frac{13}{2}, x = -2, x = 2$

Test intervals:

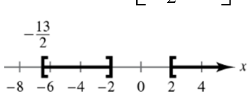
$\left(-\infty, -\frac{13}{2}\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$\left(-\frac{13}{2}, -2\right) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

$(-2, 2) \Rightarrow 2x^3 + 13x^2 - 8x - 52 < 0$

$(2, \infty) \Rightarrow 2x^3 + 13x^2 - 8x - 52 > 0$

Solution set: $\left[-\frac{13}{2}, -2\right], [2, \infty)$



31.
$$4x^3 - 6x^2 < 0$$
$$2x^2(2x - 3) < 0$$

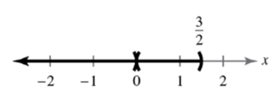
Key numbers: $x = 0, x = \frac{3}{2}$

Test intervals: $(-\infty, 0) \Rightarrow 2x^2(2x - 3) < 0$

$\left(0, \frac{3}{2}\right) \Rightarrow 2 \Rightarrow 2x^2(2x - 3) < 0$

$\left(\frac{3}{2}, \infty\right) \Rightarrow 2x^2(2x - 3) > 0$

Solution set: $(-\infty, 0) \cup \left(0, \frac{3}{2}\right)$



32.
$$x^3 - 4x \ge 0$$
$$x(x + 2)(x - 2) \ge 0$$

Key numbers: $x = 0, x = \pm 2$

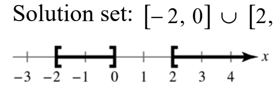
Test intervals: $(-\infty, -2) \Rightarrow x(x + 2)(x - 2) < 0$

$(-2, 0) \Rightarrow x(x + 2)(x - 2) > 0$

$(0, 2) \Rightarrow x(x + 2)(x - 2) < 0$

$(2, \infty) \Rightarrow x(x + 2)(x - 2) > 0$

Solution set: $[-2, 0] \cup [2, \infty)$



33. $(x-1)^2(x+2)^3 \geq 0$

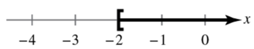
Key numbers: $x = 1, x = -2$

Test intervals: $(-\infty, -2) \Rightarrow (x-1)^2(x+2)^3 < 0$

$(-2, 1) \Rightarrow (x-1)^2(x+2)^3 > 0$

$(1, \infty) \Rightarrow (x-1)^2(x+2)^3 > 0$

Solution set: $[-2, \infty)$



34. $x^4(x-3) \leq 0$

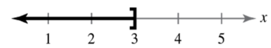
Key numbers: $x = 0, x = 3$

Test intervals: $(-\infty, 0) \Rightarrow x^4(x-3) < 0$

$(0, 3) \Rightarrow x^4(x-3) < 0$

$(3, \infty) \Rightarrow x^4(x-3) > 0$

Solution set: $(-\infty, 3]$



35. $4x^2 - 4x + 1 \leq 0$

$(2x-1)^2 \leq 0$

Key number: $x = \frac{1}{2}$

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \frac{1}{2})$	$x = 0$	$[2(0) - 1]^2 = 1$	Positive
$(\frac{1}{2}, \infty)$	$x = 1$	$[2(1) - 1]^2 = 1$	Positive

The solution set consists of the single real number $\frac{1}{2}$.

36. $x^2 + 3x + 8 > 0$

Using the Quadratic Formula you can determine the key numbers are $x = -\dfrac{3}{2} \pm \dfrac{\sqrt{23}}{2}i$.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 + 3(0) + 8 = 8$	Positive

The solution set is the set of all real numbers.

37. $x^2 - 6x + 12 \leq 0$

Using the Quadratic Formula, you can determine that the key numbers are $x = 3 \pm \sqrt{3}i$.

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 - 6(0) + 12 = 12$	Positive

The solution set is empty, that is there are no real solutions.

38. $x^2 - 8x + 16 > 0$

$(x-4)^2 > 0$

Key number: $x = 4$

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, 4)$	$x = 0$	$(0 - 4)^2 = 16$	Positive
$(4, \infty)$	$x = 5$	$(5 - 4)^2 = 1$	Positive

The solution set consists of all real numbers except $x = 4$, or $(-\infty, 4) \cup (4, \infty)$.

39. $\dfrac{4x - 1}{x} > 0$

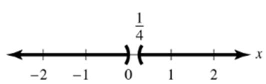
Key numbers: $x = 0, x = \dfrac{1}{4}$

Test intervals: $(-\infty, 0), \left(0, \frac{1}{4}\right), \left(\frac{1}{4}, \infty\right)$

Test: Is $\dfrac{4x - 1}{x} > 0$?

Interval	x-Value	Value of $\frac{4x-1}{x}$	Conclusion
$(-\infty, 0)$	-1	$\frac{-5}{-1} = 5$	Positive
$\left(0, \frac{1}{4}\right)$	$\frac{1}{8}$	$\frac{-\frac{1}{2}}{\frac{1}{8}} = -4$	Negative
$\left(\frac{1}{4}, \infty\right)$	1	$\frac{3}{1} = 3$	Positive

Solution set: $(-\infty, 0) \cup \left(\dfrac{1}{4}, \infty\right)$



40. $\dfrac{x - 1}{x} < 0$

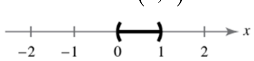
Key numbers: $0, 1$

Test intervals: $(-\infty, 0), (0, 1), (1, \infty)$

Test: Is $\dfrac{x - 1}{x} < 0$?

Interval	x-Value	Value of $\frac{x-1}{x}$	Conclusion
$(-\infty, 0)$	-1	$\frac{-1 - 1}{-1} = 2$	Positive
$(0, 1)$	$\frac{1}{2}$	$\frac{\frac{1}{2} - 1}{\frac{1}{2}} = -1$	Negative
$(1, \infty)$	2	$\frac{2 - 1}{2} = \frac{1}{2}$	Positive

Solution set: $(0, 1)$



41.
$$\frac{3x + 5}{x - 1} < 2$$
$$\frac{3x + 5}{x - 1} - 2 < 0$$
$$\frac{3x + 5 - 2(x - 1)}{x - 1} < 0$$
$$\frac{x + 7}{x - 1} < 0$$

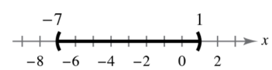
Key numbers: $x = -7, x = 1$

Test intervals: $(-\infty, -7), (-7, 1), (1, \infty)$

Test: Is $\dfrac{x + 7}{x - 1} < 0$?

Interval	x-Value	Value of $\frac{x+7}{x-1}$	Conclusion
$(-\infty, -7)$	-8	$\frac{-1}{-9} = \frac{1}{9}$	Positive
$(-7, 1)$	0	$\frac{0 + 7}{0 - 1} = -7$	Negative
$(1, \infty)$	2	$\frac{2 + 9}{2 - 1} = 11$	Positive

Solution set: $(-7, 1)$



42.
$$\frac{x + 12}{x + 2} \ge 3$$
$$\frac{x + 12}{x + 2} - 3 \ge 0$$
$$\frac{x + 12 - 3(x + 2)}{x + 2} \ge 0$$
$$\frac{6 - 2x}{x + 2} \ge 0$$

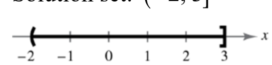
Key numbers: $x = -2, x = 3$

Test intervals: $(-\infty, -2), (-2, 3), (-3, \infty)$

Test: Is $\dfrac{6 - 2x}{x + 2} > 0$?

Interval	x-Value	Value of $\frac{6-2x}{x+2}$	Conclusion
$(-\infty, -2)$	-3	$\frac{6 - 2(-3)}{(-3) + 2} = -12$	Negative
$(-2, 3)$	0	$\frac{6 - 0}{0 + 2} = 3$	Positive
$(3, \infty)$	4	$\frac{6 - 8}{4 + 2} = \frac{1}{3}$	Negative

Solution set: $(-2, 3]$



43.

$$\frac{2}{x+5} > \frac{1}{x-3}$$

$$\frac{2}{x+5} - \frac{1}{x-3} > 0$$

$$\frac{2(x-3) - 1(x+5)}{(x+5)(x-3)} > 0$$

$$\frac{x-11}{(x+5)(x-3)} > 0$$

Key numbers: $x = -5, x = 3, x = 11$

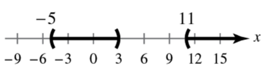
Test intervals: $(-\infty, -5) \Rightarrow \frac{x-11}{(x+5)(x-3)} < 0$

$(-5, 3) \Rightarrow \frac{x-11}{(x+5)(x-3)} > 0$

$(3, 11) \Rightarrow \frac{x-11}{(x+5)(x-3)} < 0$

$(11, \infty) \Rightarrow \frac{x-11}{(x+5)(x-3)} > 0$

Solution set: $(-5, 3) \cup (11, \infty)$



44.

$$\frac{5}{x-6} > \frac{3}{x+2}$$

$$\frac{5(x+2) - 3(x-6)}{(x-6)(x+2)} > 0$$

$$\frac{2x+28}{(x-6)(x+2)} > 0$$

Key numbers: $x = -14, x = -2, x = 6$

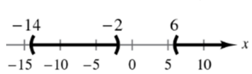
Test intervals: $(-\infty, -14) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(-14, -2) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

$(-2, 6) \Rightarrow \frac{2x+28}{(x-6)(x+2)} < 0$

$(6, \infty) \Rightarrow \frac{2x+28}{(x-6)(x+2)} > 0$

Solution intervals: $(-14, -2) \cup (6, \infty)$



45.

$$\frac{x^2 + 2x}{x^2 - 9} \le 0$$

$$\frac{x(x+2)}{(x+3)(x-3)} \le 0$$

Key numbers: $x = 0, x = -2, x = \pm 3$

Test intervals: $(-\infty, -3) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

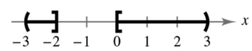
$(-3, -2) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} < 0$

$(-2, 0) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

$(0, 3) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} < 0$

$(3, \infty) \Rightarrow \frac{x(x+2)}{(x+3)(x-3)} > 0$

Solution set: $(-3, -2] \cup [0, 3)$



46.

$$\frac{x^2 + x - 6}{x^2 - 4x} \ge 0$$

$$\frac{(x+3)(x-2)}{x(x-4)} \ge 0$$

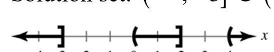
Key numbers: $-3, 0, 2, 4$

Test intervals: $(-\infty, -3), (-3, 0), (0, 2), (2, 4), (4, \infty)$

Test: Is $\frac{(x+3)(x-2)}{x(x-4)} \ge 0$?

Interval	x-Value	Value of $\frac{(x+3)(x-2)}{x(x-4)}$	Conclusion
$(-\infty, -3)$	–4	$\frac{(-1)(-6)}{(-4)(-8)} = \frac{3}{16}$	Positive
$(-3, 0)$	−1	$\frac{(2)(-3)}{(-1)(-5)} = -\frac{6}{5}$	Negative
$(0, 2)$	1	$\frac{(4)(-1)}{(1)(-3)} = \frac{4}{3}$	Positive
$(2, 4)$	3	$\frac{(6)(1)}{(3)(-1)} = -2$	Negative
$(4, \infty)$	5	$\frac{(8)(3)}{(5)(1)} = \frac{24}{5}$	Positive

Solution set: $(-\infty, -3] \cup (0, 2] \cup (4, \infty)$



47. $$\frac{3}{x-1} + \frac{2x}{x+1} > -1$$

$$\frac{3(x+1) + 2x(x-1) + 1(x+1)(x-1)}{(x-1)(x+1)} > 0$$

$$\frac{3x^2 + x + 2}{(x-1)(x+1)} > 0$$

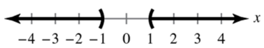
Key numbers: $x = -1, x = 1$

Test intervals: $(-\infty, -1) \Rightarrow \frac{3x^2 + x + 2}{(x-1)(x+1)} > 0$

$(-1, 1) \Rightarrow \frac{3x^2 + x + 2}{(x-1)(x+1)} < 0$

$(1, \infty) \Rightarrow \frac{3x^2 + x + 2}{(x-1)(x+1)} > 0$

Solution set: $(-\infty, -1) \cup (1, \infty)$



48. $$\frac{3x}{x-1} \le \frac{x}{x+4} + 3$$

$$\frac{3x(x+4) - x(x-1) - 3(x+4)(x-1)}{(x-1)(x+4)} \le 0$$

$$\frac{-x^2 + 4x + 12}{(x-1)(x+4)} \le 0$$

$$\frac{-(x-6)(x+2)}{(x-1)(x+4)} \le 0$$

Key numbers: $x = -4, x = -2, x = 1, x = 6$

Test intervals: $(-\infty, -4) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

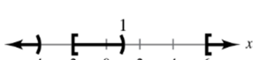
$(-4, -2) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} > 0$

$(-2, 1) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

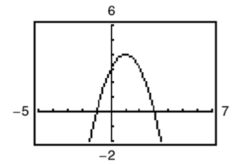
$(1, 6) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} > 0$

$(6, \infty) \Rightarrow \frac{-(x-6)(x+2)}{(x-1)(x+4)} < 0$

Solution set: $(-\infty, -4) \cup [-2, 1) \cup [6, \infty)$



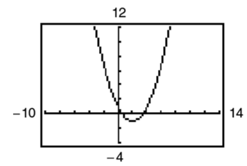
49. $y = -x^2 + 2x + 3$



(a) $y \le 0$ when $x \le -1$ or $x \ge 3$.

(b) $y \ge 3$ when $0 \le x \le 2$.

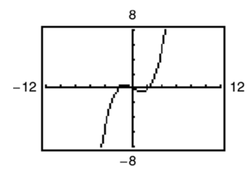
50. $y = \frac{1}{2}x^2 - 2x + 1$



(a) $y \le 0$ when $2 - \sqrt{2} \le x \le 2 + \sqrt{2}$.

(b) $y \ge 7$ when $x \le -2$ or $x \ge 6$.

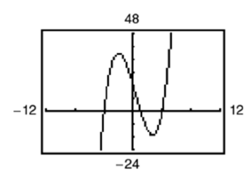
51. $y = \frac{1}{8}x^3 - \frac{1}{2}x$



(a) $y \ge 0$ when $-2 \le x \le 0$ or $2 \le x < \infty$.

(b) $y \le 6$ when $x \le 4$.

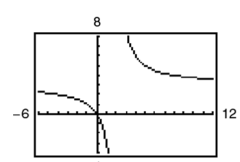
52. $y = x^3 - x^2 - 16x + 16$



(a) $y \le 0$ when $-\infty < x \le -4$ or $1 \le x \le 4$.

(b) $y \ge 36$ when $x = -2$ or $5 \le x < \infty$.

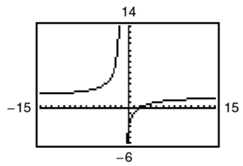
53. $y = \frac{3x}{x-2}$



(a) $y \le 0$ when $0 \le x < 2$.

(b) $y \ge 6$ when $2 < x \le 4$.

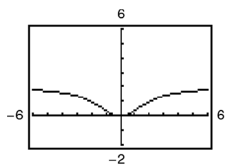
54. $y = \dfrac{2(x - 2)}{x + 1}$



(a) $y \le 0$ when $-1 < x \le 2$.

(b) $y \ge 8$ when $-2 \le x < -1$.

55. $y = \dfrac{2x^2}{x^2 + 4}$



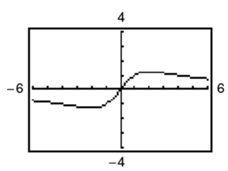
(a) $y \ge 1$ when $x \le -2$ or $x \ge 2$.

This can also be expressed as $|x| \ge 2$.

(b) $y \le 2$ for all real numbers x.

This can also be expressed as $-\infty < x < \infty$.

56. $y = \dfrac{5x}{x^2 + 4}$



(a) $y \ge 1$ when $1 \le x \le 4$.

(b) $y \le 0$ when $-\infty < x \le 0$.

57. $0.3x^2 + 6.26 < 10.8$

$0.3x^2 + 4.54 < 0$

Key numbers: $x \approx \pm 3.89$

Test intervals: $(-\infty, -3.89), (-3.89, 3.89), (3.89, \infty)$

Solution set: $(-3.89, 3.89)$

58. $-1.3x^2 + 3.78 > 2.12$

$-1.3x^2 + 1.66 > 0$

Key numbers: $x \approx \pm 1.13$

Test intervals: $(-\infty, -1.13), (-1.13, 1.13), (1.13, \infty)$

Solution set: $(-1.13, 1.13)$

59. $12.5x + 1.6 > 0.5x^2$

$-0.5x^2 + 12.5x + 1.6 > 0$

Key numbers: $x \approx -0.13, x \approx 25.13$

Test intervals: $(-\infty, -0.13), (-0.13, 25.13), (25.13, \infty)$

Solution set: $(-0.13, 25.13)$

60. $1.2x^2 + 4.8x + 3.1 < 5.3$

$1.2x^2 + 4.8x - 2.2 < 0$

Key numbers: $x \approx -4.42, x \approx 0.42$

Test intervals: $(-\infty, -4.42), (-4.42, 0.42), (0.42, \infty)$

Solution set: $(-4.42, 0.42)$

61. $\dfrac{1}{2.3x - 5.2} > 3.4$

$\dfrac{1}{2.3x - 5.2} - 3.4 > 0$

$\dfrac{1 - 3.4(2.3x - 5.2)}{2.3x - 5.2} > 0$

$\dfrac{-7.82x + 18.68}{2.3x - 5.2} > 0$

Key numbers: $x \approx 2.39, x \approx 2.26$

Test intervals: $(-\infty, 2.26), (2.26, 2.39), (2.39, \infty)$

Solution set: $(2.26, 2.39)$

62. $\dfrac{2}{3.1x - 3.7} > 5.8$

$\dfrac{2 - 5.8(3.1x - 3.7)}{3.1x - 3.7} > 0$

$\dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

Key numbers: $x \approx 1.19, x \approx 1.30$

Test intervals: $(-\infty, 1.19) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

$(1.19, 1.30) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

$(1.30, \infty) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

Solution set: $(1.19, 1.30)$

63. $s = -16t^2 + v_0t + s_0 = -16t^2 + 160t$

(a) $-16t^2 + 160t = 0$

$-16t(t - 10) = 0$

$t = 0, t = 10$

It will be back on the ground in 10 seconds.

(b) $-16t^2 + 160t > 384$

$-16t^2 + 160t - 384 > 0$

$-16(t^2 - 10t + 24) > 0$

$t^2 - 10t + 24 < 0$

$(t - 4)(t - 6) < 0$

Key numbers: $t = 4, t = 6$

Test intervals: $(-\infty, 4), (4, 6), (6, \infty)$

Solution set: 4 seconds $< t <$ 6 seconds

64. $s = -16t^2 + v_0t + s_0 = -16t^2 + 128t$

(a) $-16t^2 + 128t = 0$

$-16t(t - 8) = 0$

$-16t = 0 \Rightarrow t = 0$

$t - 8 = 0 \Rightarrow t = 8$

It will be back on the ground in 8 seconds.

(b) $-16t^2 + 128t < 128$

$-16t^2 + 128t - 128 < 0$

$-16(t^2 - 8t + 8) < 0$

$t^2 - 8t + 8 > 0$

Key numbers: $t = 4 - 2\sqrt{2}, t = 4 + 2\sqrt{2}$

Test intervals:
$(-\infty, 4 - 2\sqrt{2}), (4 - 2\sqrt{2}, 4 + 2\sqrt{2})$,
$(4 + 2\sqrt{2}, \infty)$

Solution set: 0 seconds $\leq t < 4 - 2\sqrt{2}$ seconds
and
$4 + 2\sqrt{2}$ seconds $< t \leq 8$ seconds

65. $R = x(75 - 0.0005x)$ and $C = 30x + 250{,}000$

$P = R - C$

$= (75x - 0.0005x^2) - (30x + 250{,}000)$

$= -0.0005x^2 + 45x - 250{,}000$

$P \geq 750{,}000$

$-0.0005x^2 + 45x - 250{,}000 \geq 750{,}000$

$-0.0005x^2 + 45x - 1{,}000{,}000 \geq 0$

Key numbers: $x = 40{,}000, x = 50{,}000$

(These were obtained by using the Quadratic Formula.)

Test intervals:
$(0, 40{,}000), (40{,}000, 50{,}000), (50{,}000, \infty)$

The solution set is $[40{,}000, 50{,}000]$ or $40{,}000 \leq x \leq 50{,}000$. The price per unit is

$p = \frac{R}{x} = 75 - 0.0005x.$

For $x = 40{,}000$, $p = \$55$. For $x = 50{,}000$, $p = \$50$. So, for $40{,}000 \leq x \leq 50{,}000$, $\$50.00 \leq p \leq \$55.00$.

66. $R = x(50 - 0.0002x)$ and $C = 12x + 150{,}000$

$P = R - C$

$= (50x - 0.0002x^2) - (12x + 150{,}000)$

$= -0.0002x^2 + 38x - 150{,}000$

$P \geq 1{,}650{,}000$

$-0.0002x^2 + 38x - 150{,}000 \geq 1{,}650{,}000$

$-0.0002x^2 + 38x - 1{,}800{,}000 \geq 0$

Key numbers: $x = 90{,}000$ and $x = 100{,}000$

Test intervals:
$(0, 90{,}000), (90{,}000, 100{,}000), (100{,}000, \infty)$

The solution set is $[90{,}000, 100{,}000]$ or $90{,}000 \leq x \leq 100{,}000$. The price per unit is

$p = \frac{R}{x} = 50 - 0.0002x.$

For $x = 90{,}000$, $p = \$32$. For $x = 100{,}000$, $p = \$30$. So, for $90{,}000 \leq x \leq 100{,}000$, $\$30 \leq p \leq \$32$.

67. $4 - x^2 \ge 0$

$(2 + x)(2 - x) \ge 0$

Key numbers: $x = \pm 2$

Test intervals: $(-\infty, -2) \Rightarrow 4 - x^2 < 0$

$(-2, 2) \Rightarrow 4 - x^2 > 0$

$(2, \infty) \Rightarrow 4 - x^2 < 0$

Domain: $[-2, 2]$

68. The domain of $\sqrt{x^2 - 9}$ can be found by solving the inequality:

$x^2 - 9 \ge 0$

$(x + 3)(x - 3) \ge 0$

Key numbers: $x = -3, x = 3$

Test intervals: $(-\infty, -3) \Rightarrow (x + 3)(x - 3) > 0$

$(-3, 3) \Rightarrow (x + 3)(x - 3) < 0$

$(3, \infty) \Rightarrow (x + 3)(x - 3) > 0$

Domain: $(-\infty, -3] \cup [3, \infty)$

69. $x^2 - 9x + 20 \ge 0$

$(x - 4)(x - 5) \ge 0$

Key numbers: $x = 4, x = 5$

Test intervals: $(-\infty, 4), (4, 5), (5, \infty)$

Interval	x-Value	Value of $(x - 4)(x - 5)$	Conclusion
$(-\infty, 4)$	0	$(-4)(-5) = 20$	Positive
$(4, 5)$	$\frac{9}{2}$	$\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{4}$	Negative
$(5, \infty)$	6	$(2)(1) = 2$	Positive

Domain: $(-\infty, 4] \cup [5, \infty)$

70. The domain of $\sqrt{49 - x^2}$ can be found by solving the inequality:

$49 - x^2 \ge 0$

$x^2 - 49 \le 0$

$(x + 7)(x - 7) \le 0$

Key numbers: $x = -7, x = 7$

Test intervals: $(-\infty, -7) \Rightarrow (x + 7)(x - 7) > 0$

$(-7, 7) \Rightarrow (x + 7)(x - 7) < 0$

$(7, \infty) \Rightarrow (x + 7)(x - 7) > 0$

Domain: $[-7, 7]$

71. $\dfrac{x}{x^2 - 2x - 35} \ge 0$

$\dfrac{x}{(x + 5)(x - 7)} \ge 0$

Key numbers: $x = 0, x = -5, x = 7$

Test intervals: $(-\infty, -5) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} < 0$

$(-5, 0) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} > 0$

$(0, 7) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} < 0$

$(7, \infty) \Rightarrow \dfrac{x}{(x + 5)(x - 7)} > 0$

Domain: $(-5, 0] \cup (7, \infty)$

72. $\dfrac{x}{x^2 - 9} \ge 0$

$\dfrac{x}{(x + 3)(x - 3)} \ge 0$

Key numbers: $x = -3, x = 0, x = 3$

Test intervals: $(-\infty, -3) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} < 0$

$(-3, 0) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} > 0$

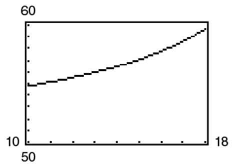
$(0, 3) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} < 0$

$(3, \infty) \Rightarrow \dfrac{x}{(x + 3)(x - 3)} > 0$

Domain: $(-3, 0] \cup (3, \infty)$

73. $S = \dfrac{52.88 - 1.89t}{1 - 0.038t}, 10 \le t \le 18$

(a)



(b)
$$S = \frac{52.88 - 1.89t}{1 - 0.038t}$$
$$57 < \frac{52.88 - 1.89t}{1 - 0.038t}$$
$$0 < \frac{52.88 - 1.89t}{1 - 0.038t} - 57$$
$$0 < \frac{-4.12 + 0.276t}{1 - 0.038t}$$

Key numbers: 14.9, 26.3

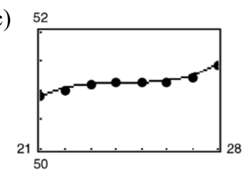
Use the domain of the model to create test intervals.

Test Interval	t-Value	Value of expression	Conclusion
$(10, 14.9)$	11	−0.76	Negative
$(14.9, 18)$	15	0.01	Positive

The mean salary was less than \$57,000 for $t < 14.9$, or during the year 2014.

(c) *Sample answer:* No. For $t < 22$, the model rapidly increases then decreases.

74. (a) and (c)



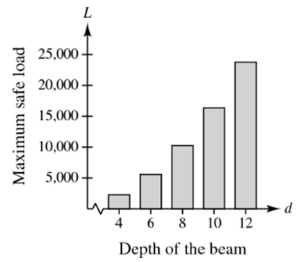
The model fits the data well.

(b) $N = 0.00624t^3 - 0.4552t^2 + 11.072t - 38.66$

(d) Using the zoom and trace features, the number of students enrolled in public elementary and secondary schools exceeded 51.20 million for $t > 26.4$, or in the year 2026.

75. (a)

d	4	6	8	10	12
Load	2223.9	5593.9	10,312	16,378	23,792



(b)
$$2000 \le 168.5d^2 - 472.1$$
$$2472.1 \le 168.5d^2$$
$$14.67 \le d^2$$
$$3.83 \le d$$

The minimum depth is 3.83 inches.

76.
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{2}$$
$$2R_1 = 2R + RR_1$$
$$2R_1 = R(2 + R_1)$$
$$\frac{2R_1}{2 + R_1} = R$$

Because $R \ge 1$,
$$\frac{2R_1}{2 + R_1} \ge 1$$
$$\frac{2R_1}{2 + R_1} - 1 \ge 0$$
$$\frac{R_1 - 2}{2 + R_1} \ge 0.$$

Because $R_1 > 0$, the only key number is $R_1 = 2$. The inequality is satisfied when $R_1 \ge 2$ ohms.

77. $2L + 2W = 100 \Rightarrow W = 50 - L$
$$LW \ge 500$$
$$L(50 - L) \ge 500$$
$$-L^2 + 50L - 500 \ge 0$$

By the Quadratic Formula you have:

Key numbers: $L = 25 \pm 5\sqrt{5}$

Test: Is $-L^2 + 50L - 500 \ge 0$?

Solution set: $25 - 5\sqrt{5} \le L \le 25 + 5\sqrt{5}$

13.8 meters $\le L \le$ 36.2 meters

78. $2L + 2W = 440 \Rightarrow W = 220 - L$

$$LW \geq 8000$$

$$L(220 - L) \geq 8000$$

$$-L^2 + 220L - 8000 \geq 0$$

By the Quadratic Formula we have:

Key numbers: $L = 110 \pm 10\sqrt{41}$

Test: Is $-L^2 + 220L - 8000 \geq 0$?

Solution set: $110 - 10\sqrt{41} \leq L \leq 110 + 10\sqrt{41}$

$$45.97 \text{ feet} \leq L \leq 174.03 \text{ feet}$$

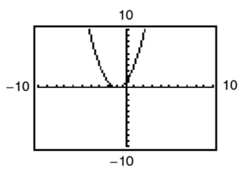
79. False.

There are four test intervals. The test intervals are $(-\infty, -3), (-3, 1), (1, 4),$ and $(4, \infty)$.

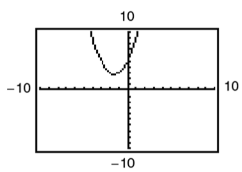
80. True.

The y-values are greater than zero for all values of x.

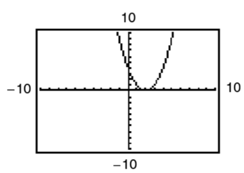
81.



For part (b), the y-values that are less than or equal to 0 occur only at $x = -1$.



For part (c), there are no y-values that are less than 0.



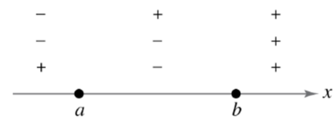
For part (d), the y-values that are greater than 0 occur for all values of x except 2.

82. Answers will vary. The key numbers of a rational inequality include the values for which it is undefined.

83. $\dfrac{1}{x}$ is undefined when $x = 0$. The correct solution set is $(0, \infty)$.

84. (a) $x = a, x = b$

(b)



(c) The real zeros of the polynomial.

85. $x^2 + bx + 9 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$$b^2 - 4(1)(9) \geq 0$$

$$b^2 - 36 \geq 0$$

Key numbers: $b = -6, b = 6$

Test intervals: $(-\infty, -6) \Rightarrow b^2 - 36 > 0$

$(-6, 6) \Rightarrow b^2 - 36 < 0$

$(6, \infty) \Rightarrow b^2 - 36 > 0$

Solution set: $(-\infty, -6] \cup [6, \infty)$

(b) $b^2 - 4ac \geq 0$

Key numbers: $b = -2\sqrt{ac}, b = 2\sqrt{ac}$

Similar to part (a), if $a > 0$ and $c > 0$, $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$.

86. $x^2 + bx - 4 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$$b^2 - 4(1)(-4) \geq 0$$
$$b^2 + 16 \geq 0$$

Key numbers: none

Test intervals: $(-\infty, \infty) \Rightarrow b^2 + 16 > 0$

Solution set: $(-\infty, \infty)$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c < 0$, b can be any real number.

87. $3x^2 + bx + 10 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$$b^2 - 4(3)(10) \geq 0$$
$$b^2 - 120 \geq 0$$

Key numbers: $b = -2\sqrt{30}, b = 2\sqrt{30}$

Test intervals: $(-\infty, -2\sqrt{30}) \Rightarrow b^2 - 120 > 0$

$(-2\sqrt{30}, 2\sqrt{30}) \Rightarrow b^2 - 120 < 0$

$(2\sqrt{30}, \infty) \Rightarrow b^2 - 120 > 0$

Solution set: $(-\infty, -2\sqrt{30}] \cup [2\sqrt{30}, \infty)$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c > 0$, $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$.

88. $2x^2 + bx + 5 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$$b^2 - 4(2)(5) \geq 0$$
$$b^2 - 40 \geq 0$$

Key numbers: $b = -2\sqrt{10}, b = 2\sqrt{10}$

Test intervals: $(-\infty, -2\sqrt{10}) \Rightarrow b^2 - 40 > 0$

$(-2\sqrt{10}, 2\sqrt{10}) \Rightarrow b^2 - 40 < 0$

$(2\sqrt{10}, \infty) \Rightarrow b^2 - 40 > 0$

Solution set: $(-\infty, -2\sqrt{10}] \cup [2\sqrt{10}, \infty)$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c > 0$, $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$.

89. $\dfrac{5 - 7}{12 - 18} = \dfrac{-2}{-6} = \dfrac{1}{3}$

90. $\dfrac{16 - 6}{6 - 11} = \dfrac{10}{-5} = -2$

91. $\dfrac{3 - 3}{4 - 0} = \dfrac{0}{4} = 0$

92. $\dfrac{1 - (-1)}{(9 - 9)} = \dfrac{2}{0} \rightarrow$ Division by 0 is undefined.

93. x-intercept: $(-1, 0)$

y-intercept: $(0, 1)$

94. x-intercept: $(-2, 0)$

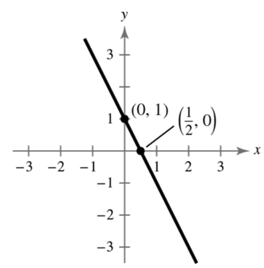
y-intercept: $(0, -4)$

95. $2x + y = 1$

$2(-x) + y = 1 \Rightarrow -2x + y = 1 \Rightarrow$ No y-axis symmetry

$2x + (-y) = 1 \Rightarrow 2x - y = 1 \Rightarrow$ No x-axis symmetry

$2(-x) + (-y) = 1 \Rightarrow -2x - y = 1 \Rightarrow$ No origin symmetry



x-intercept: $\left(\frac{1}{2}, 0\right)$

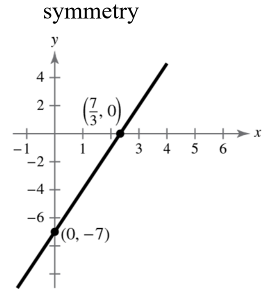
y-intercept: $(0, 1)$

96. $3x - y = 7$

$3(-x) - y = 7 \Rightarrow -3x - y = 7 \Rightarrow$ No y-axis symmetry

$3x - (-y) = 7 \Rightarrow 3x + y = 7 \Rightarrow$ No x-axis symmetry

$3(-x) - (-y) = 7 \Rightarrow -3x + y = 7 \Rightarrow$ No origin symmetry

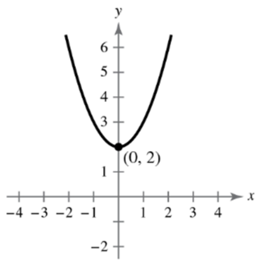


97. $y = x^2 + 2$

$y = (-x)^2 + 2 \Rightarrow y = x^2 + 2 \Rightarrow$ y-axis symmetry

$-y = x^2 + 2 \Rightarrow y = -x^2 - 2 \Rightarrow$ No x-axis symmetry

$-y = (-x)^2 + 2 \Rightarrow y = -x^2 - 2 \Rightarrow$ No origin symmetry



No x-intercepts

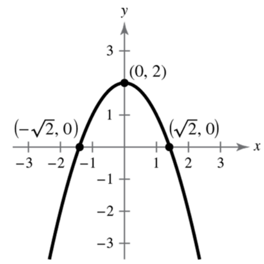
y-intercept: $(0, 2)$

98. $y = 2 - x^2$

$y = 2 - (-x)^2 \Rightarrow y = 2 - x^2 \Rightarrow$ y-axis symmetry

$-y = 2 - x^2 \Rightarrow y = -2 + x^2 \Rightarrow$ No x-axis symmetry

$-y = 2 - (-x)^2 \Rightarrow y = -2 + x^2 \Rightarrow$ No origin symmetry



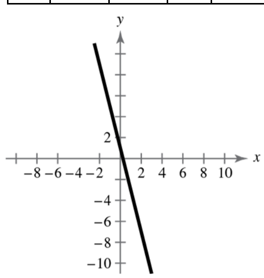
x-intercepts: $(\pm\sqrt{2}, 0)$

y-intercept: $(0, 2)$

Review Exercises for Chapter 1

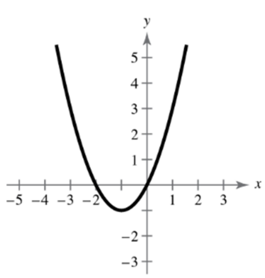
1. $y = -4x + 1$

x	-2	-1	0	1	2
y	9	5	1	-3	-7



2. $y = x^2 + 2x$

x	-3	-2	-1	0	1
y	3	0	-1	0	3



3. x-intercepts: $(-4, 0), (2, 0)$

y-intercept: $(0, -2)$

4. x-intercepts: $(1, 0), (5, 0)$

y-intercept: $(0, 5)$

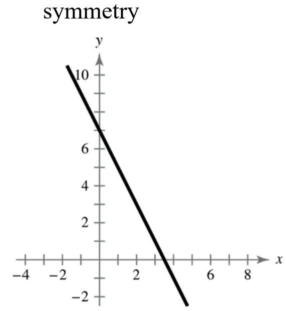
5. $y = -3x + 7$

Intercepts: $\left(\frac{7}{3}, 0\right), (0, 7)$

$y = -3(-x) + 7 \Rightarrow y = 3x + 7 \Rightarrow$ No y-axis symmetry

$-y = -3x + 7 \Rightarrow y = 3x - 7 \Rightarrow$ No x-axis symmetry

$-y = -3(-x) + 7 \Rightarrow y = -3x - 7 \Rightarrow$ No origin symmetry



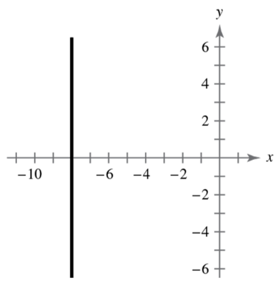
6. $x = -8$

Intercept: $(-8, 0)$, No y-intercept.

$-x = -8 \Rightarrow x = 8 \Rightarrow$ No y-axis symmetry

$x = -8 \Rightarrow$ x-axis symmetry

$-x = -8 \Rightarrow x = 8 \Rightarrow$ No origin symmetry



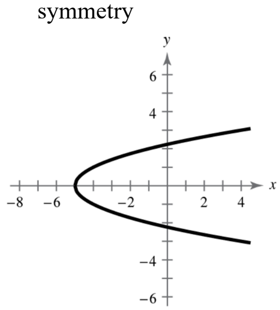
7. $x = y^2 - 5$

Intercepts: $(-5, 0)$, $\left(0, \pm\sqrt{5}\right)$

$-x = y^2 - 5 \Rightarrow x = -y^2 + 5 \Rightarrow$ No y-axis symmetry

$x = \left(-y^2\right) - 5 \Rightarrow x = y^2 - 5 \Rightarrow$ x-axis symmetry

$-x = (-y)^2 - 5 \Rightarrow x = -y^2 + 5 \Rightarrow$ No origin symmetry



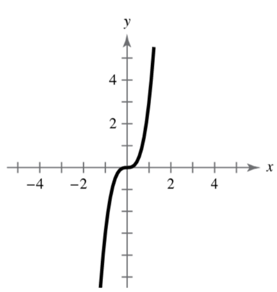
8. $y = 3x^3$

Intercepts: $(0, 0)$

$y = 3(-x)^3 \Rightarrow y = -3x^3 \Rightarrow$ No y-axis symmetry

$-y = 3x^3 \Rightarrow y = -3x^3 \Rightarrow$ No x-axis symmetry

$-y = 3(-x)^3 \Rightarrow y = 3x^3 \Rightarrow$ Original symmetry



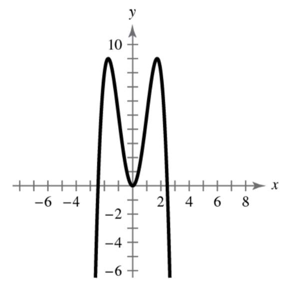
9. $y = -x^4 + 6x^2$

Intercept: $(0, 0)$

$y = -(-x)^4 + 6(-x)^2 \Rightarrow y = -x^4 + 6x^2 \Rightarrow$ y-axis symmetry

$-y = -x^4 + 6x^2 \Rightarrow y = x^4 - 6x^2 \Rightarrow$ No x-axis symmetry

$-y = -(-x)^4 + 6(-x)^2 \Rightarrow y = x^4 - 6x^2 \Rightarrow$ No origin symmetry



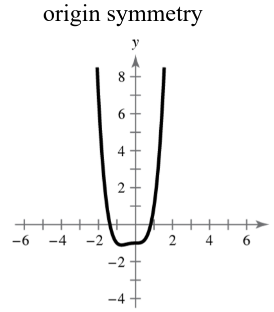
10. $y = x^4 + x^3 - 1$

Intercept: $(0, -1)$

$y = (-x)^4 + (-x)^3 - 1 \Rightarrow y = x^4 - x^3 - 1 \Rightarrow$ y-axis symmetry

$-y = x^4 + x^3 - 1 \Rightarrow y = -x^4 - x^3 + 1 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 + (-x)^3 - 1 \Rightarrow y = x^4 - x^3 - 1 \Rightarrow$ No origin symmetry



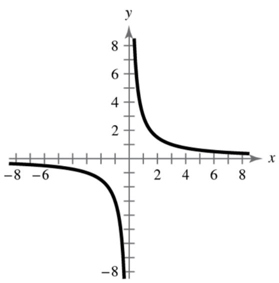
11. $y = \dfrac{3}{x}$

Intercept: None

$y = \dfrac{3}{-x} \Rightarrow y = -\dfrac{3}{x} \Rightarrow$ No y-axis symmetry

$-y = \dfrac{3}{x} \Rightarrow y = -\dfrac{3}{x} \Rightarrow$ No x-axis symmetry

$-y = \dfrac{3}{-x} \Rightarrow y = \dfrac{3}{x} \Rightarrow$ origin symmetry



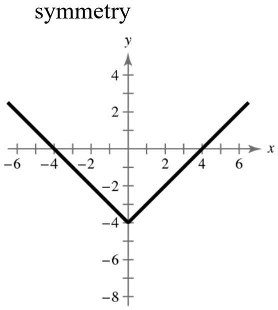
12. $y = |x| - 4$

Intercepts: $(\pm 4, 0), (0, -4)$

$y = |-x| - 4 \Rightarrow y = |x| - 4 \Rightarrow y$-axis symmetry

$-y = |x| - 4 \Rightarrow y = -|x| + 4 \Rightarrow$ No x-axis symmetry

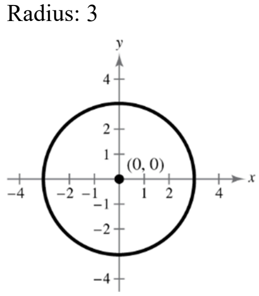
$-y = |-x| - 4 \Rightarrow y = -|x| + 4 \Rightarrow$ No origin symmetry



13. $x^2 + y^2 = 9$

Center: $(0, 0)$

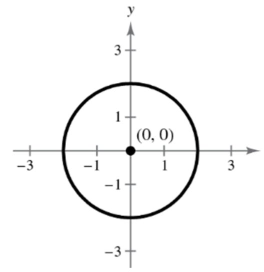
Radius: 3



14. $x^2 + y^2 = 4$

Center: $(0, 0)$

Radius: 2

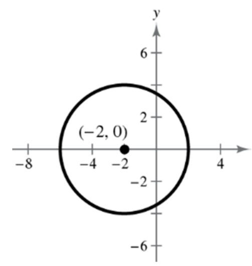


15. $(x + 2)^2 + y^2 = 16$

$(x - (-2))^2 + (y - 0)^2 = 4^2$

Center: $(-2, 0)$

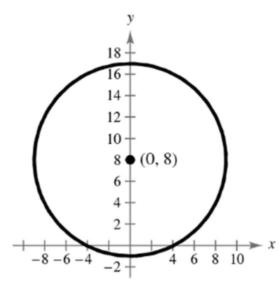
Radius: 4



16. $x^2 + (y - 8)^2 = 81$

Center: $(0, 8)$

Radius: 9



17. Endpoints of a diameter: $(0, 0)$ and $(4, -6)$

Center: $\left(\dfrac{0 + 4}{2}, \dfrac{0 + (-6)}{2}\right) = (2, -3)$

Radius: $r = \sqrt{(2 - 0)^2 + (-3 - 0)^2} = \sqrt{4 + 9}$

$= \sqrt{13}$

Standard form: $(x - 2)^2 + (y - (-3))^2 = \left(\sqrt{13}\right)^2$

$(x - 2)^2 + (y + 3)^2 = 13$

18. Endpoints of a diameter: $(-2, -3)$ and $(4, -10)$

Center: $\left(\frac{-2+4}{2}, \frac{-3+(-10)}{2}\right) = \left(1, -\frac{13}{2}\right)$

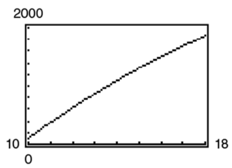
Radius: $r = \sqrt{(1-(-2))^2 + \left(-\frac{13}{2} - (-3)\right)^2} = \sqrt{9 + \frac{49}{4}} = \sqrt{\frac{85}{4}}$

Standard form: $(x-1)^2 + \left(y - \left(-\frac{13}{2}\right)\right)^2 = \left(\sqrt{\frac{85}{4}}\right)^2$

$$(x-1)^2 + \left(y + \frac{13}{2}\right)^2 = \frac{85}{4}$$

19. $S = -6.876t^2 + 411.94t - 3329.1,\ 10 \le t \le 18$

(a)



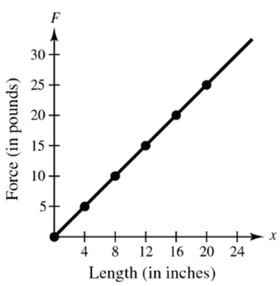
(b) Sales were \$1 billion when $t \approx 13.6$, or the year 2013.

20. $F = \frac{5}{4}x,\ 0 \le x \le 20$

(a)

x	0	4	8	12	16	20
F	0	5	10	15	20	25

(b)



(c) When $x = 10, F = \frac{50}{4} = 12.5$ pounds.

21. $2(x-2) = 2x - 4$

$2x - 4 = 2x - 4$

$0 = 0$ Identity

All real numbers are solutions.

22. $2(x+3) = 2x - 2$

$2x + 6 = 2x - 2$

$6 = -2$ Contradiction

No solution

23. $3(x-2) + 2x = 2(x+3)$

$3x - 6 + 2x = 2x + 6$

$3x = 12$

$x = 4$

Conditional equation

24. $5(x-1) - 2x = 3x - 5$

$5x - 5 - 2x = 3x - 5$

$3x - 5 = 3x - 5$

$0 = 0$ Identity

All real numbers are solutions.

25. $8x - 5 = 3x + 20$

$5x = 25$

$x = 5$

26. $7x + 3 = 3x - 17$

$4x = -20$

$x = -5$

27. $2(x+5) - 7 = x + 9$

$2x + 10 - 7 = x + 9$

$2x + 3 = x + 9$

$x = 6$

28.
$$\begin{aligned} 7(x-4) &= 1-(x+9) \\ 7x-28 &= 1-x-9 \\ 7x-28 &= -x-8 \\ 8x &= 20 \\ x &= \tfrac{5}{2} \end{aligned}$$

29.
$$\begin{aligned} \frac{x}{5}-3 &= \frac{x}{3}+1 \\ 15\left(\frac{x}{5}-3\right) &= \left(\frac{x}{3}+1\right)15 \\ 3x-45 &= 5x+15 \\ -2x &= 60 \\ x &= -30 \end{aligned}$$

30.
$$\begin{aligned} \frac{4x-3}{6}+\frac{x}{4} &= x-2 \\ 2(4x-3)+3x &= 12x-24 \\ 8x-6+3x &= 12x-24 \\ -x &= -18 \\ x &= 18 \end{aligned}$$

31.
$$\begin{aligned} 3+\frac{2}{x-5} &= \frac{2x}{x-5} \\ \frac{3(x-5)+2}{x-5} &= \frac{2x}{x-5} \\ \frac{3x-15+2}{x-5} &= \frac{2x}{x-5} \\ \frac{3x-13}{x-5} &= \frac{2x}{x-5} \\ (x-5)\frac{3x-13}{x-5} &= \frac{2x}{x-5}(x-5) \\ 3x-13 &= 2x \\ x &= 3 \end{aligned}$$

32.
$$\begin{aligned} \frac{1}{x^2+3x-18}-\frac{3}{x+6} &= \frac{4}{x-3} \\ \frac{1}{(x+6)(x-3)}-\frac{3(x-3)}{(x+6)(x-3)} &= \frac{4(x+6)}{(x+6)(x-3)} \\ (x+6)(x-3)\left[\frac{1}{(x+6)(x-3)}-\frac{3(x-3)}{(x+6)(x-3)}\right] &= \frac{4(x+6)}{(x+6)(x-3)}(x+6)(x-3) \\ 1-3(x-3) &= 4(x+6) \\ 1-3x+9 &= 4x+24 \\ -7x &= 14 \\ x &= -2 \end{aligned}$$

33. $y = 3x - 1$

x-intercept: $0 = 3x - 1 \Rightarrow x = \frac{1}{3}$

y-intercept: $y = 3(0) - 1 \Rightarrow y = -1$

The x-intercept is $\left(\frac{1}{3}, 0\right)$ and the y-intercept is $(0, -1)$.

34.
$$\begin{aligned} y &= -5x+6 \\ 0 &= -5x+6 \\ -6 &= -5x \\ \tfrac{6}{5} &= x \end{aligned} \qquad \begin{aligned} y &= -5x+6 \\ y &= -5(0)+6 \\ y &= 6 \end{aligned}$$

The x-intercept is $\left(\frac{6}{5}, 0\right)$ and the y-intercept is $(0, 6)$.

35. $y = 2(x - 4)$

x-intercept: $0 = 2(x - 4) \Rightarrow x = 4$

y-intercept: $y = 2(0 - 4) \Rightarrow y = -8$

The x-intercept is $(4, 0)$ and the y-intercept is $(0, -8)$.

36.
$$\begin{aligned} y &= 4(7x+1) \\ 0 &= 4(7x+1) \\ 0 &= 28x+4 \\ -4 &= 28x \\ -\tfrac{1}{7} &= x \end{aligned} \qquad \begin{aligned} y &= 4(7x+1) \\ y &= 4[7(0)+1] \\ y &= 4 \end{aligned}$$

The x-intercept is $\left(-\frac{1}{7}, 0\right)$ and the y-intercept is $(0, 4)$.

37. $y = -\frac{1}{2}x + \frac{2}{3}$

x-intercept: $0 = -\frac{1}{2}x + \frac{2}{3} \Rightarrow x = \frac{2/3}{1/2} = \frac{4}{3}$

y-intercept: $y = -\frac{1}{2}(0) + \frac{2}{3} \Rightarrow y = \frac{2}{3}$

The x-intercept is $\left(\frac{4}{3}, 0\right)$ and the y-intercept is $\left(0, \frac{2}{3}\right)$.

38.
$$y = \tfrac{3}{4}x - \tfrac{1}{4} \qquad y = \tfrac{3}{4}x - \tfrac{1}{4}$$
$$0 = \tfrac{3}{4}x - \tfrac{1}{4} \qquad y = \tfrac{3}{4}(0) - \tfrac{1}{4}$$
$$\tfrac{4}{3} \cdot \tfrac{1}{4} = \tfrac{4}{3} \cdot \tfrac{3}{4}x \qquad y = -\tfrac{1}{4}$$
$$\tfrac{1}{3} = x$$

The x-intercept is $\left(\tfrac{1}{3}, 0\right)$ and the y-intercept is $\left(0, -\tfrac{1}{4}\right)$.

39.
$$244.92 = 2(3.14)(3)^2 + 2(3.14)(3)h$$
$$244.92 = 56.52 + 18.84h$$
$$188.40 = 18.84h$$
$$10 = h$$

The height is 10 inches.

40.
$$C = \tfrac{5}{9}F - \tfrac{160}{9}$$
$$\tfrac{5}{9}F = C + \tfrac{160}{9}$$
$$F = \tfrac{9}{5}\left(C + \tfrac{160}{9}\right)$$

For $C = 100°$, $F = \tfrac{9}{5}\left(100 + \tfrac{160}{9}\right) = 212°$F.

41. Let x be the revenue for 2018.

The revenue for 2019 is $x + 0.283x$.

The total revenue is 12.1.
$$x + (x + 0.283x) = 12.1$$
$$2.283x = 12.1$$
$$x = \frac{12.1}{2.283} \approx 5.30$$

Revenue for 2018: \$5.30 billion

Revenue for 2019: $5.30 + 0.283(5.30) = \$6.80$ billion

42. *Model:* $(\text{Original price}) = \dfrac{(\text{sale price})}{(1 - \text{discount rate})}$

Labels: Original price $= x$

Discount rate $= 0.2$

Sale price $= 340$

Equation:
$$x = \frac{340}{1 - 0.2}$$
$$x = 425$$

The original price was \$425.

43. Let $x =$ the total investment required.

Each person's share is $\dfrac{x}{9}$. If three more people invest, each person's share is $\dfrac{x}{12} + 2500$.

Since this is \$2500 less than the original cost, we have:
$$\frac{x}{9} = \frac{x}{12} + 2500$$
$$\frac{x}{9} - \frac{x}{12} = 2500$$
$$\frac{8x - 6x}{72} = 2500$$
$$\frac{2x}{72} = 2500$$
$$x = 90{,}000$$

The total investment to start the business is \$90,000.

44.

	Rate	Time	Distance
To work	r	$\frac{56}{r}$	56
From work	$r + 8$	$\frac{56}{r+8}$	56

$$\text{Time} = \frac{\text{distance}}{\text{rate}}$$

Time to work = time from work + 10 minutes

$$\frac{56}{r} = \frac{56}{r+8} + \frac{1}{6} \quad \text{Convert minutes to portion of an hour.}$$

$$6(r + 8)(56) = 6r(56) + r(r + 8)$$

$$336r + 2688 = 336r + r^2 + 8r$$

$$0 = r^2 + 8r - 2688$$

$$0 = (r - 48)(r + 56)$$

Using the positive value for r, we have $r = 48$ miles per hour. The average speed on the trip home was $r + 8 = 56$ miles per hour.

45. Let x = the number of liters of pure antifreeze.

$$30\% \text{ of } (10 - x) + 100\% \text{ of } x = 50\% \text{ of } 10$$

$$0.30(10 - x) + 1.00x = 0.50(10)$$

$$3 - 0.30x + 1.00x = 5$$

$$0.70x = 2$$

$$x = \frac{2}{0.70} = \frac{20}{7} = 2.857 \text{ liters}$$

46. *Model:* $\left(\text{Interest from } 4\tfrac{1}{2}\%\right) + \left(\text{Interest from } 5\tfrac{1}{2}\%\right) = (\text{total interest})$

Labels: Amount invested at $4\frac{1}{2}\% = x$, amount invested at $5\frac{1}{2}\% = 6000 - x$

Interest from $4\frac{1}{2}\% = x(0.045)(1)$, interest from $5\frac{1}{2}\% = (6000 - x)(0.055)(1)$, total interest = \$3.05

Equation: $0.045x + 0.055(6000 - x) = 305$

$$0.045x + 330 - 0.055x = 305$$

$$-0.01x = -25$$

$$x = 2500$$

The amount invested at $4\frac{1}{2}\%$ was \$2500 and the amount invested at $5\frac{1}{2}\%$ was $6000 - 2500 = \$3500$.

47. $V = \frac{1}{3}\pi r^2 h$

$$3V = \pi r^2 h$$

$$\frac{3V}{\pi r^2} = h$$

48. $E = \frac{1}{2}mv^2$

$$mv^2 = 2E$$

$$m = \frac{2E}{v^2}$$

49. $15 + x - 2x^2 = 0$

$$0 = 2x^2 - x - 15$$

$$0 = (2x + 5)(x - 3)$$

$$2x + 5 = 0 \Rightarrow x = -\tfrac{5}{2}$$

$$x - 3 = 0 \Rightarrow x = 3$$

50. $2x^2 - x - 28 = 0$

$(2x + 7)(x - 4) = 0$

$2x + 7 = 0 \Rightarrow x = -\frac{7}{2}$

$x - 4 = 0 \Rightarrow x = 4$

51. $6 = 3x^2$

$2 = x^2$

$\pm\sqrt{2} = x$

52. $16x^2 = 25$

$16x^2 - 25 = 0$

$(4x - 5)(4x + 5) = 0$

$4x - 5 = 0 \Rightarrow x = \frac{5}{4}$

$4x + 5 = 0 \Rightarrow x = -\frac{5}{4}$

53. $(x + 13)^2 = 25$

$x + 13 = \pm 5$

$x = -13 \pm 5$

$x = -18$ or $x = -8$

54. $(x - 5)^2 = 30$

$x - 5 = \pm\sqrt{30}$

$x = 5 \pm \sqrt{30}$

55. $x^2 + 12x + 25 = 0$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(1)(25)}}{2(1)}$$

$$= \frac{-12 \pm 2\sqrt{11}}{2}$$

$$= -6 \pm \sqrt{11}$$

56. $9x^2 - 12x = 14$

$9x^2 - 12x - 14 = 0$

$$x = \frac{-12 \pm \sqrt{(-12)^2 - 4(9)(-14)}}{2(9)}$$

$$= \frac{-12 \pm 18\sqrt{2}}{18}$$

$$= \frac{2}{3} \pm \sqrt{2}$$

57. $-2x^2 - 5x + 27 = 0$

$2x^2 + 5x - 27 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-27)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{241}}{4}$$

58. $-20 - 3x + 3x^2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

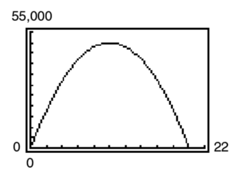
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-20)}}{2(3)}$$

$$= \frac{3 \pm \sqrt{249}}{6} = \frac{1}{2} \pm \frac{\sqrt{249}}{6}$$

59. $M = 500x(20 - x)$

(a) $500x(20 - x) = 0$ when $x = 0$ feet and $x = 20$ feet.

(b)



(c) The bending moment is greatest when $x = 10$ feet.

60. (a) $h(t) = -16t^2 + 30t + 5.8$

(b) $h(1) = -16 \cdot 1^2 + 30 \cdot 1 + 5.8 = 19.8$ feet

(c) $-16t^2 + 30t + 5.8 = 6.2$

$-16t^2 + 30t - 0.4 - 0$

$$t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(-0.4)}}{2(-16)}$$

$$= \frac{-30 \pm \sqrt{874.4}}{-32}$$

≈ 1.86157 or 0.01343

The ball will hit the ground in about 1.86 seconds.

61. $\sqrt{-18}\sqrt{-6} = (\sqrt{18}i)(\sqrt{6}i) = \sqrt{108}(i^2) = -6\sqrt{3}$

62. $\sqrt{-27} + \sqrt{-3} = 3\sqrt{3}i + \sqrt{3}i = 4\sqrt{3}i$

63. $(5 + \sqrt{-10})(10 - \sqrt{-5}) = (5 + \sqrt{10}i)(10 - \sqrt{5}i)$

$= 50 + 10\sqrt{10}i - 5\sqrt{5}i - \sqrt{50}(i^2)$

$= 50 + \sqrt{50}i + 10\sqrt{10}i - 5\sqrt{5}i$

$= 50 + 5\sqrt{2} + (10\sqrt{10} - 5\sqrt{5})i$

64. $\left(6 - \sqrt{-2}\right)^2 = \left(6 - \sqrt{2}i\right)\left(6 - \sqrt{2}i\right)$

$= 36 - 6\sqrt{2}i - 6\sqrt{2}i + \left(\sqrt{2}i\right)^2$

$= 36 - 2 - 6\sqrt{2}i - 6\sqrt{2}i$

$= 34 - 12\sqrt{2}i$

65. $(6 - 4i) + (-9 + i) = \left(6 + (-9)\right) + (-4i + i) = -3 - 3i$

66. $(7 - 2i) - (3 - 8i) = (7 - 3) + (-2i + 8i) = 4 + 6i$

67. $-3i(-2 + 5i) = 6i - 15i^2$

$= 6i - 15(-1)$

$= 15 + 6i$

68. $(4 + i)(3 - 10i) = 12 - 40i + 3i - 10i^2$

$= 12 - 37i - 10(-1)$

$= 22 - 37i$

69. $(1 + 7i)(1 - 7i) = 1 - 49i^2$

$= 1 - 49(-1)$

$= 1 + 49$

$= 50$

70. $(5 - 9i)^2 = 25 - 90i + 81i^2$

$= 25 - 90i + 81(-1)$

$= 25 - 81 - 90i$

$= -56 - 90i$

71. $\dfrac{4}{1 - 2i} = \dfrac{4}{1 - 2i} \cdot \dfrac{1 + 2i}{1 + 2i}$

$= \dfrac{4 + 8i}{1 - 4i^2}$

$= \dfrac{4 + 8i}{5}$

$= \dfrac{4}{5} + \dfrac{8}{5}i$

72. $\dfrac{6 - 5i}{i} = \dfrac{6 - 5i}{i} \cdot \dfrac{-i}{-i}$

$= \dfrac{-6i + 5i^2}{-i^2}$

$= -5 - 6i$

73. $\dfrac{3 + 2i}{5 + i} = \dfrac{3 + 2i}{5 + i} \cdot \dfrac{5 - i}{5 - i}$

$= \dfrac{15 - 3i + 10i - 2i^2}{25 - i^2}$

$= \dfrac{17 + 7i}{26}$

$= \dfrac{17}{26} + \dfrac{7i}{26}$

74. $\dfrac{7i}{(3 + 2i)^2} = \dfrac{7i}{9 + 12i + 4i^2}$

$= \dfrac{7i}{9 + 12i + 4(-1)}$

$= \dfrac{7i}{5 + 12i}$

$= \dfrac{7i}{5 + 12i} \cdot \dfrac{5 - 12i}{5 - 12i}$

$= \dfrac{35i - 84i^2}{25 - 144i^2}$

$= \dfrac{35i - 84(-1)}{25 + 144}$

$= \dfrac{84 + 35i}{169}$

$= \dfrac{84}{169} + \dfrac{35}{169}i$

75. $\dfrac{4}{2 - 3i} + \dfrac{2}{1 + i} = \dfrac{4}{2 - 3i} \cdot \dfrac{2 + 3i}{2 + 3i} + \dfrac{2}{1 + i} \cdot \dfrac{1 - i}{1 - i}$

$= \dfrac{8 + 12i}{4 + 9} + \dfrac{2 - 2i}{1 + 1}$

$= \dfrac{8}{13} + \dfrac{12}{13}i + 1 - i$

$= \left(\dfrac{8}{13} + 1\right) + \left(\dfrac{12}{13}i - i\right)$

$= \dfrac{21}{13} - \dfrac{1}{13}i$

76. $\frac{1}{2+i} - \frac{5}{1+4i} = \frac{(1+4i) - 5(2+i)}{(2+i)(1+4i)}$

$= \frac{1 + 4i - 10 - 5i}{2 + 81 + i + 4i^2}$

$= \frac{-9 - i}{-2 + 9i} \cdot \frac{(-2 - 9i)}{(-2 - 9i)}$

$= \frac{18 + 81i + 2i + 9i^2}{4 - 81i^2}$

$= \frac{9 + 83i}{85} = \frac{9}{85} + \frac{83i}{85}$

77. $x^2 - 2x + 10 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)}$

$= \frac{2 \pm \sqrt{-36}}{2}$

$= \frac{2 \pm 6i}{2}$

$= 1 \pm 3i$

78. $x^2 + 6x + 34 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-6 \pm \sqrt{(6)^2 - 4(1)(34)}}{2(1)}$

$= \frac{-6 \pm \sqrt{-100}}{2}$

$= \frac{-6 \pm 10i}{2}$

$= -3 \pm 5i$

79. $4x^2 + 4x + 7 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{(4)^2 - 4(4)(7)}}{2(4)}$

$= \frac{-4 \pm \sqrt{-96}}{8}$

$= \frac{-4 \pm 4\sqrt{6}i}{8}$

$= -\frac{1}{2} \pm \frac{\sqrt{6}}{2}i$

80. $6x^2 + 3x + 27 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-3 \pm \sqrt{3^2 - 4(6)(27)}}{2(6)}$

$= \frac{-3 \pm \sqrt{-639}}{12}$

$= \frac{-3 \pm 3i\sqrt{71}}{12} = -\frac{1}{4} \pm \frac{\sqrt{71}}{4}i$

81. $5x^4 - 12x^3 = 0$

$x^3(5x - 12) = 0$

$x^3 = 0$ or $5x - 12 = 0$

$x = 0$ or $x = \frac{12}{5}$

82. $4x^3 - 6x^2 = 0$

$x^2(4x - 6) = 0$

$x^2 = 0 \Rightarrow x = 0$

$4x - 6 = 0 \Rightarrow x = \frac{3}{2}$

83. $x^3 - 7x^2 + 4x = 28$

$x^2(x - 7) + 4(x - 7) = 0$

$(x - 7)(x^2 + 4) = 0$

$x - 7 = 0 \Rightarrow x = 7$

$x^2 + 4 = 0 \Rightarrow x^2 = -4$

$x = \pm\sqrt{-4} = \pm 2i$

84. $9x^4 + 27x^3 - 4x^2 = 12x$

$9x^3(x + 3) - 4x(x + 3) = 0$

$(x + 3)(9x^3 - 4x) = 0$

$(x + 3)(x)(9x^2 - 4) = 0$

$x + 3 = 0 \Rightarrow x = -3$

$x = 0$

$9x^2 - 4 = 0 \Rightarrow x = \pm\frac{2}{3}$

85.

$$x^6 - 7x^3 - 8 = 0$$
$$(x^3)^2 - 7(x^3) - 8 = 0$$
$$u^2 - 7u - 8 = 0$$
$$(u - 8)(u + 1) = 0$$
$$u - 8 = 0$$
$$x^3 - 8 = 0$$
$$(x - 2)(x^2 + 2x + 4) = 0$$
$$x - 2 = 0 \Rightarrow x = 2$$
$$x^2 + 2x + 4 = 0 \Rightarrow x = -1 \pm \sqrt{3}i$$
$$u + 1 = 0$$
$$x^3 + 1 = 0$$
$$(x + 1)(x^2 - x + 1) = 0$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x^2 - x + 1 = 0 \Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

86.

$$x^4 - 13x^2 - 48 = 0$$
$$(x^2)^2 - 13(x^2) - 48 = 0$$

Let $u = x^2$.

$$u^2 - 13u - 48 = 0$$
$$(u - 16)(u + 3) = 0$$
$$u - 16 = 0 \Rightarrow u = 16$$
$$u + 3 = 0 \Rightarrow u = -3$$

$u = 16 \quad u = -3$

$x^2 = 16 \quad x^2 = -3$

$x = \pm 4 \quad x = \pm\sqrt{3}i$

87.

$$\sqrt{2x + 3} = 2 + x$$
$$2x + 3 = (2 + x)^2$$
$$2x + 3 = 4 + 4x + x^2$$
$$x^2 + 2x + 1 = 0$$
$$(x + 1)^2 = 0$$
$$x = -1$$

88.

$$5\sqrt{x} - \sqrt{x - 1} = 6$$
$$5\sqrt{x} = 6 + \sqrt{x - 1}$$
$$25x = 36 + 12\sqrt{x - 1} + x - 1$$
$$24x - 35 = 12\sqrt{x - 1}$$
$$576x^2 - 1680x + 1225 = 144(x - 1)$$
$$576x^2 - 1824x + 1369 = 0$$
$$x = \frac{-(-1824) \pm \sqrt{(-1824)^2 - 4(576)(1369)}}{2(576)} = \frac{1824 \pm \sqrt{172{,}800}}{1152} = \frac{1824 \pm 240\sqrt{3}}{1152}$$
$$x = \frac{38 + 5\sqrt{3}}{24}$$
$$x = \frac{38 - 5\sqrt{3}}{24}, \text{ extraneous}$$

89.

$$(x - 1)^{2/3} - 25 = 0$$
$$(x - 1)^{2/3} = 25$$
$$(x - 1)^2 = 25^3$$
$$x - 1 = \pm\sqrt{25^3}$$
$$x = 1 \pm 125$$
$$x = 126 \text{ or } x = -124$$

90.

$$(x + 2)^{3/4} = 27$$
$$x + 2 = 27^{4/3}$$
$$x + 2 = 81$$
$$x = 79$$

91.
$$\frac{5}{x} = 1 + \frac{3}{x+2}$$
$$5(x+2) = 1(x)(x+2) + 3x$$
$$5x + 10 = x^2 + 2x + 3x$$
$$10 = x^2$$
$$\pm\sqrt{10} = x$$

92.
$$\frac{6}{x} + \frac{8}{x+5} = 3$$
$$x(x+5)\frac{6}{x} + x(x+5)\frac{8}{x+5} = 3x(x+5)$$
$$6(x+5) + 8x = 3x(x+5)$$
$$14x + 30 = 3x^2 + 15x$$
$$0 = 3x^2 + x - 30$$
$$0 = (3x+10)(x-3)$$
$$0 = 3x + 10 \Rightarrow x = -\frac{10}{3}$$
$$0 = x - 3 \Rightarrow x = 3$$

93. $|x - 5| = 10$

$x - 5 = -10$ or $x - 5 = 10$

$x = -5$ $\qquad$ $x = 15$

94. $|2x + 3| = 7$

$|2x + 3| = 7$ or $2x + 3 = -7$

$2x = 4$ $\qquad$ $2x = -10$

$x = 2$ $\qquad$ $x = -5$

95. $|x^2 - 3| = 2x$

$x^2 - 3 = 2x$ or $x^2 - 3 = -2x$

$x^2 - 2x - 3 = 0$ $\qquad$ $x^2 + 2x - 3 = 0$

$(x-3)(x+1) = 0$ $\qquad$ $(x+3)(x-1) = 0$

$x = 3$ or $x = -1$ $\qquad$ $x = -3$ or $x = 1$

The only solutions of the original equation are $x = 3$ or $x = 1$. ($x = 3$ and $x = -1$ are extraneous.)

96. $|x^2 - 6| = x$

$x^2 - 6 = x$ or $-(x^2 - 6) = x$

$x^2 - x - 6 = 0$ $\qquad$ $x^2 + x - 6 = 0$

$(x-3)(x+2) = 0$ $\qquad$ $(x+3)(x-2) = 0$

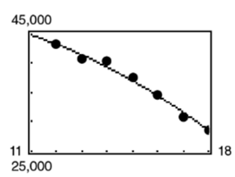
$x - 3 = 0 \Rightarrow x = 3$ $\qquad$ $x - 2 = 0 \Rightarrow x = 2$

$x + 2 = 0 \Rightarrow x = -2$, extraneous $\qquad$ $x + 3 = 0 \Rightarrow x = -3$, extraneous

97.
$$29.95 = 42 - \sqrt{0.001x + 2}$$
$$-12.05 = -\sqrt{0.001x + 2}$$
$$\sqrt{0.001x + 2} = 12.05$$
$$0.001x + 2 = 145.2025$$
$$0.001x = 143.2025$$
$$x = 143{,}202.5$$
$$\approx 143{,}203 \text{ units}$$

98. $C = 51; 663 - 16.772t^{5/2}, 11 \le t \le 18$

(a)



The model fits the data well.

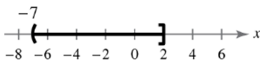
(b) Paid circulation was about 37 million daily newspapers when $t \approx 15.25$, or in the year 2015.

(c)
$$37{,}000 = 51{,}663 - 16.772t^{5/2}$$
$$16.772t^{5/2} = 14{,}663$$
$$t^{5/2} \approx 874.25$$
$$t \approx 15.02$$

Paid circulation was about 37 million daily newspapers when $t \approx 15.02$, or in the year 2015.

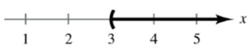
99. Interval: $(-7, 2]$; The interval is bounded.

Inequality: $-7 < x \le 2$



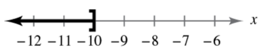
100. Interval: $(3, \infty)$; The interval is unbounded.

Inequality: $x > 3$



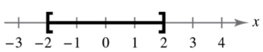
101. Interval: $(-\infty, -10]$; The interval is unbounded.

Inequality: $x \le -10$



102. Interval: $[-2, 2]$; The interval is bounded.

Inequality: $-2 \le x \le 2$

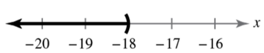


103. $3(x + 2) < 2x - 12$

$3x + 6 < 2x - 12$

$x < -18$

$(-\infty\ -18)$



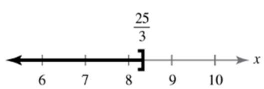
104. $2(x + 5) \ge 5(x - 3)$

$2x + 10 \ge 5x - 15$

$-3x \ge -25$

$x \le \frac{25}{3}$

$\left(-\infty, \frac{25}{3}\right]$



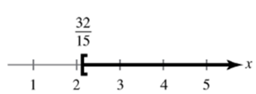
105. $4(5 - 2x) \le \frac{1}{2}(8 - x)$

$20 - 8x \le 4 - \frac{1}{2}x$

$-\frac{15}{2}x \le -16$

$x \ge \frac{32}{15}$

$\left[\frac{32}{15}, \infty\right)$



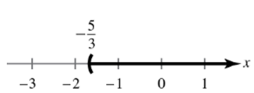
106. $\frac{1}{2}(3 - x) > \frac{1}{3}(2 - 3x)$

$9 - 3x > 4 - 6x$

$3x > -5$

$x > -\frac{5}{3}$

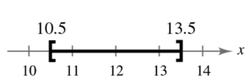
$\left(-\frac{5}{3}, \infty\right)$



107. $3.2 \le 0.4x - 1 \le 4.4$

$4.2 \le 0.4x \le 5.4$

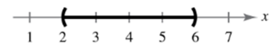
$10.5 \le x \le 13.5$



108. $1.6 < 0.3x + 1 < 2.8$

$0.6 < 0.3x < 1.8$

$2 < x < 6$

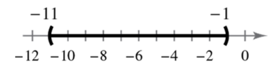


109. $|x + 6| < 5$

$-5 < x + 6 < 5$

$-11 < x < -1$

$(-11, -1)$



110. $\frac{2}{3}|3 - x| \ge 4$

$|3 - x| \ge 6$

$3 - x \le -6$ or $3 - x \ge 6$

$-x \le -9$ or $-x \ge 3$

$x \ge 9$ or $x \le -3$

$x \le -3$ or $x \ge 9$

$(-\infty, -3] \cup [9, \infty)$

111. $125.33x > 92x + 1200$

$33.33x > 1200$

$x > 36$ units

So, the smallest value of x for which the product returns a profit is 37 units.

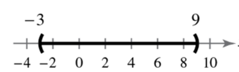
112. If the side is 19.3 cm, then with the possible error of 0.5 cm we have:

$18.8 \le \text{side} \le 19.8$

$353.44\ \text{cm}^2 \le \text{area} \le 392.04\ \text{cm}^2$

113. $x^2 - 6x - 27 < 0$

$(x + 3)(x - 9) < 0$



Key numbers: $x = -3, x = 9$

Test intervals: $(-\infty, -3), (-3, 9), (9, \infty)$

Test: Is $(x + 3)(x - 9) < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-3, 9)$.

114. $x^2 - 2x \ge 3$

$x^2 - 2x - 3 \ge 0$

$(x - 3)(x + 1) \ge 0$

Key numbers: $x = -1, x = 3$

Test intervals: $(-\infty, -1), (-1, 3), (3, \infty)$

Test: Is $(x - 3)(x + 1) \ge 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty, -1] \cup [3, \infty)$.

115. $5x^3 - 45x < 0$

$5x(x^2 - 9) < 0$

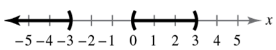
$5x(x + 3)(x - 3) < 0$

Key numbers: $x = \pm 3, x = 0$

Test intervals: $(-\infty, -3), (-3, 0), (0, 3), (3, \infty)$

Test: Is $5x(x + 3)(x - 3) < 0$?

By testing an x-value in each test interval in the inequality, the solution set is $(-\infty, -3) \cup (0, 3)$.



116. $2x^3 - 5x^2 - 3x \geq 0$

$x(2x^2 - 5x - 3) \geq 0$

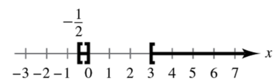
$x(2x + 1)(x - 3) \geq 0$

Key numbers: $x = 0 - \frac{1}{2}, 3$

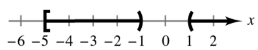
Test intervals: $\left(-\infty, -\frac{1}{2}\right), \left(-\frac{1}{2}, 0\right), (0, 3), (3, \infty)$

Test: Is $x(2x + 1)(x - 3) \geq 0$?

By testing an x-value in each test interval in the inequality, the solution set is $\left[-\frac{1}{2}, 0\right] \cup [3, \infty)$.



117. $\dfrac{2}{x + 1} \leq \dfrac{3}{x - 1}$



$$\frac{2(x - 1) - 3(x + 1)}{(x + 1)(x - 1)} \leq 0$$

$$\frac{2x - 2 - 3x - 3}{(x + 1)(x - 1)} \leq 0$$

$$\frac{-(x + 5)}{(x + 1)(x - 1)} \leq 0$$

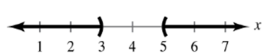
Key numbers: $x = -5, x = -1, x = 1$

Test intervals: $(-5, -1), (-1, 1), (1, \infty)$

Test: Is $\dfrac{-(x + 5)}{(x + 1)(x - 1)} \leq 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $[-5, -1) \cup (1, \infty)$.

118. $\dfrac{x - 5}{3 - x} < 0$



Key numbers: $x = 5, x = 3$

Test intervals: $(-\infty, 3), (3, 5), (5, \infty)$

Test: Is $\dfrac{x - 5}{3 - x} < 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty, 3) \cup (5, \infty)$.

119. $5000(1 + r)^2 > 5500$

$(1 + r)^2 > 1.1$

$1 + r > 1.0488$

$r > 0.0488$

$r > 4.9\%$

120. $P = \dfrac{1000(1 + 3t)}{5 + t}$

$2000 \leq \dfrac{1000(1 + 3t)}{5 + t}$

$2000(5 + t) \leq 1000(1 + 3t)$

$10{,}000 + 2000t \leq 1000 + 3000t$

$-1000t \leq -9000$

$t \geq 9$ days

At least 9 days are required.

121. False.

$\sqrt{-18}\sqrt{-2} = \left(\sqrt{18}i\right)\left(\sqrt{2}i\right) = \sqrt{36}i^2 = -6$

$\sqrt{(-8)(-2)} = \sqrt{36} = 6$

122. False. The equation has no real solution.

The solutions are

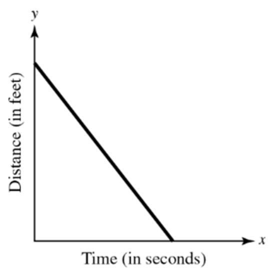
$$\frac{717}{650} \pm \frac{\sqrt{3311}i}{650}.$$

123. Rational equations, equations involving radicals, and absolute value equations, may have "solutions" that are extraneous. So checking solutions, in the original equations, is crucial to eliminate these extraneous values.

124. $\left[-\infty, -\frac{36}{11}\right]$ or $[2, \infty)$. *Sample answer:* The first equivalent inequality written is incorrect. It should be $11x + 4 \le -26$. This leads to the solution $\left(-\infty, -\frac{30}{11}\right]$ or $[2, \infty)$.

Problem Solving for Chapter 1

1.



2. (a) $1 + 2 + 3 + 4 + 5 = 15$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 36$

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

(b) $1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n + 1)$

When $n = 5$: $\frac{1}{2}(5)(6) = 15$

When $n = 8$: $\frac{1}{2}(8)(9) = 36$

When $n = 10$: $\frac{1}{2}(10)(11) = 55$

(c)
$$\frac{1}{2}n(n + 1) = 210$$
$$n(n + 1) = 420$$
$$n^2 + n - 420 = 0$$
$$(n + 21)(n - 20) = 0$$
$$n = -21 \text{ or } n = 20$$

Since n is a natural number, choose $n = 20$.

3. (a) $A = \pi ab$

$a + b = 20 \Rightarrow b = 20 - a$, thus:

$A = \pi a(20 - a)$

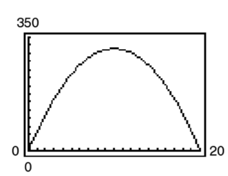
(b)

a	4	7	10	13	16
A	64π	91π	100π	91π	64π

(c)
$$300 = \pi a(20 - a)$$
$$300 = 20\pi a - \pi a^2$$
$$\pi a^2 - 20\pi a + 300 = 0$$
$$a = \frac{20\pi \pm \sqrt{(-20\pi)^2 - 4\pi(300)}}{2\pi}$$
$$= \frac{20\pi \pm \sqrt{400\pi^2 - 1200\pi}}{2\pi}$$
$$= \frac{20\pi \pm 20\sqrt{\pi(\pi - 3)}}{2\pi}$$
$$= 10 \pm \frac{10}{\pi}\sqrt{\pi(\pi - 3)}$$

$a \approx 12.12$ or $a \approx 7.88$

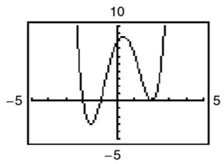
(d)



(e) The a-intercepts occur at $a = 0$ and $a = 20$. Both yield an area of 0. When $a = 0$, $b = 20$ and you have a vertical line of length 40. Likewise when $a = 20$, $b = 0$ and you have a horizontal line of length 40. They represent the minimum and maximum values of a.

(f) The maximum value of A is $100\pi \approx 314.159$. This occurs when $a = b = 10$ and the ellipse is actually a circle.

4. $y = x^4 - x^3 - 6x^2 + 4x + 8 = (x - 2)^2(x + 1)(x + 2)$



From the graph you see that $x^4 - x^3 - 6x^2 + 4x + 8 > 0$ on the intervals $(-\infty, -2) \cup (-1, 2) \cup (2, \infty)$.

5. $P = 0.00256s^2$

(a) $0.00256s^2 = 20$

$s^2 = 7812.5$

$s \approx 88.4$ miles per hour

(b) $0.00256s^2 = 40$

$s^2 = 15625$

$s = 125$ miles per hour

No, actually it can survive wind blowing at $\sqrt{2}$ times the speed found in part (a).

(c) The wind speed in the formula is squared, so a small increase in wind speed could have potentially serious effects on a building.

6. $h = \left(\sqrt{h_0} - \dfrac{2\pi d^2\sqrt{3}}{lw}t\right)^2$

$l = 60'', w = 30'', h_0 = 25'', d = 2''$

$$h = \left(5 - \frac{8\pi\sqrt{3}}{1800}t\right)^2 = \left(5 - \frac{\pi\sqrt{3}}{225}t\right)^2$$

(a) $12.5 = \left(5 - \dfrac{\pi\sqrt{3}}{225}t\right)^2$

$\sqrt{12.5} = 5 - \dfrac{\pi\sqrt{3}}{225}t$

$t = \dfrac{225}{\pi\sqrt{3}}\left(5 - \sqrt{12.5}\right) \approx 60.6$ seconds

(b) $0 = \left(\sqrt{12.5} - \dfrac{\pi\sqrt{3}}{225}t\right)^2$

$t = \dfrac{225\sqrt{12.5}}{\pi\sqrt{3}} \approx 146.2$ seconds

(c) The speed at which the water drains decreases as the amount of the water in the bathtub decreases.

7. (a) If $x^2 + 9 = (x + m)(x + n)$ then

$mn = 9$ and $m + n = 0$.

(b) $m + n = 0 \Rightarrow n = -m$

$m(-m) = 9 \Rightarrow -m^2 = 9 \Rightarrow m^2 = -9$

There is no **integer** m such that m^2 equals a negative number. $x^2 + 9$ cannot be factored over the integers.

8.

$$4\sqrt{x} = 2x + k$$

$$2x - 4\sqrt{x} + k = 0 \quad \text{Complete the square.}$$

$$x - 2\sqrt{x} = -\frac{k}{2}$$

$$x - 2\sqrt{x} + 1 = 1 - \frac{k}{2}$$

$$\left(\sqrt{x} - 1\right)^2 = 1 - \frac{k}{2}$$

Number of solutions (real)	Some k-values
2	−1, 0, 1
1	2 only
0	3, 4, 5

This equation will have two solutions when $1 - \frac{k}{2} > 0$ or when $k < 2$.

This equation will have one solution when $1 - \frac{k}{2} = 0$ or when $k = 2$.

This equation will have no solutions when $1 - \frac{k}{2} < 0$ or when $k > 2$.

9. (a) 5, 12, and 13; 8, 15, and 17

7, 24, and 25

(b) $5 \cdot 12 \cdot 13 = 780$ which is divisible by 3, 4, and 5.

$8 \cdot 15 \cdot 17 = 2040$ which is divisible by 3, 4, and 5.

$7 \cdot 24 \cdot 25 = 4200$ which is also divisible by 3, 4, and 5.

(c) Conjecture: If $a^2 + b^2 = c^2$ where a, b, and c are positive integers, then abc is divisible by 60.

10.

Equation	x_1, x_2	$x_1 + x_2$	$x_1 \cdot x_2$
(a) $x^2 - x - 12 = 0$	4, −3	1	−12
(b) $2x^2 + 5x - 3 = 0$ $(2x - 1)(x + 3) = 0$	$\frac{1}{2}, -3$	$-\frac{5}{2}$	$-\frac{3}{2}$
(c) $4x^2 - 9 = 0$ $(2x + 3)(2x - 3) = 0$	$-\frac{3}{2}, \frac{3}{2}$	0	$-\frac{9}{4}$
(d) $x^2 - 10x + 34 = 0$ $x = 5 \pm 3i$	$5 + 3i, 5 - 3i$	10	34

11. (a) $S = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} + \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-2a}{2a}$

$= -\dfrac{b}{a}$

(b) $P = \left(\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\dfrac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$

$= \dfrac{b^2 - (b^2 - 4ac)}{4a^2}$

$= \dfrac{4ac}{4a^2}$

$= \dfrac{c}{a}$

12. (a) (i) $\left(\dfrac{-5 + 5\sqrt{3}i}{2}\right)^3 = 125$

(ii) $\left(\dfrac{-5 - 5\sqrt{3}i}{2}\right)^3 = 125$

(b) (i) $\left(\dfrac{-3 + 3\sqrt{3}i}{2}\right)^3 = 27$

(ii) $\left(\dfrac{-3 - 3\sqrt{3}i}{2}\right)^3 = 27$

(c) (i) The cube roots of 1 are: $1, \dfrac{-1 \pm \sqrt{3}i}{2}$

(ii) The cube roots of 8 are: $2, -1 \pm \sqrt{3}i$

(iii) The cube roots of 64 are: $4, -2 \pm 2\sqrt{3}i$

13. (a) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{1+i} = \dfrac{1}{1+i} \cdot \dfrac{1-i}{1-i}$

$= \dfrac{1-i}{2} = \dfrac{1}{2} - \dfrac{1}{2}i$

(b) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{3-i} = \dfrac{1}{3-i} \cdot \dfrac{3+i}{3+i}$

$= \dfrac{3+i}{10} = \dfrac{3}{10} + \dfrac{1}{10}i$

(c) $z_m = \dfrac{1}{z}$

$= \dfrac{1}{-2+8i}$

$= \dfrac{1}{-2+8i} \cdot \dfrac{-2-8i}{-2-8i}$

$= \dfrac{-2-8i}{68} = -\dfrac{1}{34} - \dfrac{2}{17}i$

14. $(a + bi)(a - bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2$

Since a and b are real numbers, $a^2 + b^2$ is also a real number.

15. (a) $c = 1$

The terms are: $i, -1 + i, -i, -1 + i, -i, -1 + i, -i, -1 + i, -i, \ldots$

The sequence is bounded so $c = i$ *is* in the Mandelbrot Set.

(b) $c = -2$

The terms are: $1 + i, 1 + 3i, -7 + 7i, 1 - 97i, -9407 - 1931i, \ldots$

The sequence is unbounded so $c = 1 + i$ *is not* in the Mandelbrot Set.

(c) $c = -2$

The terms are: $-2, 2, 2, 2, 2, \ldots$

The sequence is bounded so $c = -2$ *is* in the Mandelbrot Set.

Practice Test for Chapter 1

1. Graph $3x - 5y = 15$.

2. Graph $y = \sqrt{9 - x}$.

3. Solve $5x + 4 = 7x - 8$.

4. Solve $\frac{x}{3} - 5 = \frac{x}{5} + 1$.

5. Solve $\frac{3x + 1}{6x - 7} = \frac{2}{5}$.

6. Solve $(x - 3)^2 + 4 = (x + 1)^2$.

7. Solve $A = \frac{1}{2}(a + b)h$ for a.

8. 301 is what percent of 4300?

9. Cindy has \$6.05 in quarter and nickels. How many of each coin does she have if there are 53 coins in all?

10. Ed has \$15,000 invested in two fund paying $9\frac{1}{2}\%$ and 11% simple interest, respectively. How much is invested in each if the yearly interest is \$1582.50?

11. Solve $28 + 5x - 3x^2 = 0$ by factoring.

12. Solve $(x - 2)^2 = 24$ by taking the square root of both sides.

13. Solve $x^2 - 4x - 9 = 0$ by completing the square.

14. Solve $x^2 + 5x - 1 = 0$ by the Quadratic Formula.

15. Solve $3x^2 - 2x + 4 = 0$ by the Quadratic Formula.

16. The perimeter of a rectangle is 1100 feet. Find the dimensions so that the enclosed area will be 60,000 square feet.

17. Find two consecutive even positive integers whose product is 624.

18. Solve $x^3 - 10x^2 + 24x = 0$ by factoring.

19. Solve $\sqrt[3]{6 - x} = 4$.

20. Solve $(x^2 - 8)^{2/5} = 4$.

21. Solve $x^4 - x^2 - 12 = 0$.

22. Solve $4 - 3x > 16$.

23. Solve $\left|\frac{x - 3}{2}\right| < 5$.

24. Solve $\frac{x + 1}{x - 3} < 2$.

25. Solve $|3x - 4| \geq 9$.